

# Mathematics for Business I

1<sup>st</sup> G.A.D.E., Academic Year 2011/12

## Control Unit 2 (Option A)

SURNAME(S): ..... NAME: .....

1. Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear map such that  $(1, 0, 0)$  is an eigenvector with eigenvalue  $\lambda = 3$ ,  $f(0, 1, 0) = (1, 3, 0)$  and  $f(0, 0, 1) = (2, 4, 5)$ . Let  $M(f)$  the matrix associated to  $f$  in the canonical basis.
  - a) Calculate  $M(f)$ .
  - b) Calculate the eigenvalues of  $f$ , its multiplicities, and an eigenvector of each eigenvalue (different from  $(1, 0, 0)$ ).
  - c) Study if  $f$  is diagonalizable.

**Solution:**

a)  $M(F) = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix}.$

- b) It has two eigenvalues: 3 (with multiplicity  $m_1 = 2$ ), and 5 (with multiplicity  $m_2 = 1$ ).  
 $(2, 2, 1)$  is an eigenvector of the eigenvalue 5.  $(2, 0, 0)$  is an eigenvector of the eigenvalue 3.
- c) It is not diagonalizable, because  $\dim H(3) = 1 \neq m_1$ .

2. The return of an investment,  $R$ , depends on three financial parameters  $x$ ,  $y$ ,  $z$ , through the relationship  $R(x, y, z) = x^2 + 2y^2 + (k+1)z^2 + 2kyz + 2xz$ , where  $k \in \mathbb{R}$ .

- a) Find a value for the parameter  $k$ , such that the investment is always profitable (i.e., the return always positive).
- b) Find a value for the parameter  $k$ , such that the investment is always profitable or null.
- c) Find a value for the parameter  $k$ , such that the investment is sometimes profitable and sometimes non profitable.
- d) Consider  $k = 2$  and study if the investment is profitable or not when  $y = x - z$ .
- e) Consider  $k = 2$  and study if the investment is profitable or not when  $z = x + y$  and  $x = 2y$ .

**Solution:**

- a) The minors of the matrix are  $A_1 > 0$ ,  $A_2 > 0$ ,  $A_3 = 2k - k^2 = k(2 - k)$ .  $A_3 > 0$  when  $0 < k < 2$ . So it could be  $k = 1$ , for example.
- b)  $A_3 = 0$  when  $k = 0$  or  $k = 2$ .
- c)  $A_3 < 0$  when  $k < 0$  or  $k > 2$ . So it could be  $k = 3$ , for example.
- d)  $R$  is positive definite in that case, so yes, it is profitable.
- e)  $R$  is positive definite in that case, so yes, it is profitable.