Control, option B

$\boxed{2}$ a) Number (1) is not linear,

because $f(0,0,0,0) = (0,-10) \neq \vec{0}$

b) $f(1,0,0,0) = (1, \frac{1}{2})$

$f(0,1,0,0) = (2,0)$

$f(0,0,1,0) = (0,-1)$

$f(0,0,0,1) = (3,0)$

$$M(f) = \begin{pmatrix} 1 & 2 & 0 & 3 \\ \frac{1}{2} & 0 & -1 & 0 \end{pmatrix}$$

c) $f(\vec{u}) = M(f)\cdot\vec{u} = \begin{pmatrix} 1 & 2 & 0 & 3 \\ \frac{1}{2} & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{3}{2} \end{pmatrix}$

$f(1,-1,2,0) = (-1, -\frac{3}{2})$

we can calculate it by replacing the components of $\vec{u}$ in the expressions of f, aswell:

$$f(1,-1,2,0) = \left(1+2(-1)+3\cdot 0,\ \frac{1}{2}\cdot 1-2\right) = \left(-1, -\frac{3}{2}\right)$$

d) $f(\vec{v}) = \vec{0}$

$$\left.\begin{array}{l} x + 2y + 3t = 0 \\ \frac{1}{2}x - z = 0 \end{array}\right\}$$

$$A|B = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 0 \\ \frac{1}{2} & 0 & -1 & 0 & 0 \end{array}\right)$$

rank $(A) = 2 =$ rank $(A|B) \neq$ n° variables

$\Rightarrow$ It is a indeterminate compatible system, i.e., it has infinite solutions. We calculate one:

$z = \frac{1}{2}x$. I choose, for example $x = 1$.

So $z = \frac{1}{2}$. Then I replace those values in the other equation:

$1 + 2y + 3t = 0$; $2y = 3t - 1$; $y = \frac{3t-1}{2}$

I now choose $t = 0$ (for example). Then $y = -\frac{1}{2}$

So $\vec{v} = (1, -\frac{1}{2}, \frac{1}{2}, 0)$ verifies $f(\vec{v}) = \vec{0}$.

I check the result:

$f(1, -\frac{1}{2}, \frac{1}{2}, 0) = (1 + 2(-\frac{1}{2}) + 3 \cdot 0, \frac{1}{2} \cdot 1 - \frac{1}{2}) = (0, 0)$