Mathematics for Business I

1^{st} G.A.D.E., Academic Year 2011/12

Control Unit 1, option B

SURNAME(S): ..... NAME: .....

1. In $\mathbb{R}^3$, consider the vectors $\overrightarrow{u_1} = (-2, 2, 1), \overrightarrow{u_2} = (-1, 1, \frac{1}{2}), \overrightarrow{u_3} = (0, -1, 0), \overrightarrow{u_4} = (3, 3, 3)\}$. Answer the next questions, explaining the reasoning you use for your answers.

   a) Study if the vectors $\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}, \overrightarrow{u_4}$ are linearly dependent or independent.

   b) Study if the vectors $\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}, \overrightarrow{u_4}$ are a generating system in $\mathbb{R}^3$.

   c) Are the vectors $\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}, \overrightarrow{u_4}$ a basis in $\mathbb{R}^3$? If not, use those vectors to build up a basis in $\mathbb{R}^3$ (that is to say, using those vectors, add and/or drop some vectors to build up a basis in $\mathbb{R}^3$).

   d) Let S be a vector subspace with basis $\{\overrightarrow{u_1}, \overrightarrow{u_3}\}$. Calculate $dim(S)$, the equation(s) of S and another vector in S.

   e) Let S be a vector subspace with basis $\{\overrightarrow{u_3}\}$. Calculate $dim(S)$, the equation(s) of S and another vector in S.

2. Let's consider the maps
   (1) $f: \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ such as $f(x, y, z, t) = (x + 2y + 3t, \frac{1}{2}x - z - 10)$
   (2) $f: \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ such as $f(x, y, z, t) = (x + 2y + 3t, \frac{1}{2}x - z)$.

   a) Say which one is not a linear map and why it isn't.

   For the map that it is linear, answer the next questions:

   b) Calculate its associated matrix.

   c) Calculate the image of the vector $\overrightarrow{u} = (1, -1, 2, 0)$.

   d) Calculate one vector $\overrightarrow{v} = (x, y, z, t)$ verifying $f(\overrightarrow{v}) = \overrightarrow{0}$