## UNIT 3. DERIVATIVE AND ITS APPLICATION TO BUSINESS

## EXERCISES TO BE SOLVED USING DERIVE

Note: In this file, both $L N()$ and $\log ()$ means the napierian logarithm (logaritmo neperiano) or logarithm with basis the number $e$. In Derive, you can write $L N()$ or $\log ()$, as the program will interpret the napierian logarithm in any case.

On the other hand, to write in Derive the number $e$ you cannot write it directly from the keyboard, you must click on the icon $e$ at the right bottom instead.

1. Calculate the gradient vector of the following functions.
a) $f(x, y, z)=\frac{x^{2}+y^{2}}{z(x+y)}$ at the point $\left(2,-\frac{3}{4,1}\right)$.
b) $f(x, y, z)=\frac{x y z^{5}\left(x^{5}-y^{2}-z^{2}\right)}{x^{3}+y^{2}+z^{2}}$ at the point $\left(2,-\frac{3}{4,1}\right)$.
c) $f(x, y, z)=x y \mathbf{e}^{x^{2}}+\mathbf{e}^{y} z \mathbf{e}^{x}$ at the point $\left(2,-\frac{3}{4,1}\right)$.
d) $f(x, y, z)=(\mathbf{x}+y) \mathrm{e}^{y z}$ at the point $\left(2,-\frac{3}{4,1}\right)$.
e) $f(r, s, t)=\frac{\ln (\mathbf{r}+\mathbf{t})}{\mathbf{r}+\boldsymbol{s}+\mathbf{t}}$ at the point $\left(2,-\frac{3}{4,1}\right)$.
f) $f(x, y, z, t, u)=z t^{3} \log (x+y)+x e^{x^{2} y}+\sqrt{u z}$ at any point $(x, y, z, t, u)$.
g) $f(x, y, z, t, u)=y^{4 x}+\frac{4 e z}{\sqrt{x y}}+\frac{\sin (u)}{\cos (t)}$ at any point $(x, y, z, t, u)$.
2. Calculate the gradient vector at the point $(3,-1 / 2,1)$ for the next functions:
a) $f(x, y, z)=z \cdot L N(x+y)+x \cdot e^{x^{2} y}$
b) $f(x, y, z)=y^{2} \cdot \sqrt{z}+\frac{x^{5 / 3} \cdot \cos (z \cdot x)}{e^{z}}$
c) $f(x, y, z)=x^{3} \frac{L N(4 \cdot z)}{\sin (\sqrt{x \cdot y)}}$
d) $f(x, y, z)=y^{4 x}+\frac{e(4 z)}{\sqrt{x \cdot y}}$
3. Calculate the hessian matrix of the following functions at the point $(1,1,1)$.
a) $f(x, y, z)=2 x^{2}-x y+y^{2}+2 z^{3}+6$
b) $\quad f(x, y, z)=2 x^{2} y^{2}+x^{2} z^{2}-3 x z$
c) $f(x, y, z)=e^{x z}+e^{y z}+z^{x y}$
d) $f(x, y z z)=x^{2} y^{3} z^{4}$
e) $\quad f(x, y, z)=x^{3} y^{2}+x^{2} z+x y$
f) $\mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{z})=3 y^{3} \cdot \log (z)+\sqrt{x}$
4. The benefit of a firm which produces three kinds of chocolate is given by the function $B(x, y, z)=\frac{\mathbf{x}^{2}+\mathbf{y}^{2}}{\mathbf{z}(\mathbf{x}+\mathbf{y})}$, where $x, y$ and $z$ are the quantities of chocolate $\mathrm{A}, \mathrm{B}$ and C respectively. The current production is 5,8 and 6 units respectively. The company is thinking about increasing the production of one of type of chocolate
because of Christmas. Which kind of chocolate would you suggest to increase its production?
5. The amount of units of the good $A$ that are sold in a determinate shop is given by the function $\mathbf{P}(\mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{v})=\frac{\mathbf{1 0 0 x y}}{\mathbf{x y + u v}}$, where $x$ is the temperature, $y$ the income, $u$ the price of the good A , and $v$ the intensity of the sound. Now, the temperature in the shop is $21^{\circ} \mathrm{C}$ and the music has an intensity of 70 decibels. Moreover, the item A costs $10 \mathrm{~m} . \mathrm{u}$. and the average income of the customers is $50 \mathrm{~m} . \mathrm{u}$. You are asked the following:
a) What would be the percentage of change in consumption if the temperature increases by $2 \%$ ?
b) What would be the percentage of change in consumption if the income decreases $25 \%$ ?
6. Let $U(x, y, z)=2 z \log \left[(x+2)^{2}\right]\left(\frac{y}{2}-1\right)$ be the utility function that a person has upon consuming three goods $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and $\mathrm{A}_{3}$, in quantities $x, y$ and $z$ respectively. At present, he consumes 8 units of $A_{1}, 3$ of $A_{1}$ and 2 of $A_{3}$. Use the differential calculus to estimate the approximate variation in the utility obtained if consumptions of $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ are doubled and the consumption of $\mathrm{A}_{3}$ fells by half.
7. TRACEL is a company which produces tables. At present, there are 15 employees working to produce 80 tables every month. However, this firm is going through a bad moment due to the economic crisis. For this reason, the management is thinking about reducing the staff in 16 people. Nevertheless, he does want to avoid the benefits are decreased. Those benefits are given by the function $\mathbf{f}(\mathrm{x}, \mathrm{y})=\mathrm{xy} \mathrm{e}^{2 \mathrm{xy}}+\mathrm{yx}$, where $x$ is the number of tables produced and $y$ is the number of employees. Calculate how the production should be increased in order to maintain the same benefit.
8. The function $P=20 x^{3}+\sqrt{5 y}-\frac{3 z}{2}$ expresses the pollution generated by a productive process $Q(x, y, z)=x y z$, where $x, y$ and $z$ represent the quantities used of three inputs. Currently, the firm has a level of pollution of 319 units using $\mathrm{y}=5$ and $\mathrm{z}=4$. How would an increase of 1 unit in the quantity of the first factor affect pollution and production? And an increase of 1 unit in the second factor?
9. A website, which also offers Internet access, has a income function given by the expression: $\mathbf{I}(\mathbf{x}, \mathbf{y}, \mathbf{z})=\mathbf{4 0 x}+\mathbf{1 1 0 0 z}+\mathbf{z}\left(\mathbf{1 5 y}+\mathbf{1 0 x}-\frac{\mathbf{x y}}{\mathbf{z}}\right)$, where $x$ are thousands of users, $y$ thousands of pages visited daily, and $z$ thousands of advertisements. Currently, there are 10,000 users, 20,000 pages visited and 7,000 ads. How would the income vary if the number of users decreases to 5500 , and the number of ads rises up to 9000 ?
10. The quantity of a product demanded by a consumer is given by the function $d(p, r)=\frac{5}{\sqrt{r}}\left(\frac{r \sqrt{r}}{10}+3 \log (p)\right)$, where $p$ is the price of the product and $r$ is the consumer's income. Calculate the price-elasticity and the income-elasticity of the demand at any point ( $p, r$ ). Then calculate those elasticities for $\mathrm{p}=200$ and $\mathrm{r}=2000$, and interpret the results.
11. Calculate which variable has the greatest marginal value at the point $(2,2,2,2,2)$ for the function $f(x, y, z, t, v)=(L N(x))^{2 t-4 z v}-\frac{v e^{x y t}}{t z L N\left(v e^{x y+2 t}\right)}$.
