

1) La ecuación diferencial es:  $\ddot{x}(t) = \zeta_e(t) - \zeta_s(t) = f(t) -$   
 donde  $f(t) = \begin{cases} 0 & \text{si } t < 0 \\ t/\pi & \text{si } 0 \leq t \leq \pi \\ 2-t/\pi & \text{si } \pi < t \leq 2\pi \\ 0 & \text{si } t > 2\pi \end{cases}$

$| x(0) = 0$   
Condición inicial

a) (I)  $| x_I(t) = 0 \quad \forall t < 0 |$

(II)  $x(t) = x_h(t) + x_p(t)$ ,  $x_h(t) = C e^{-t}$ , s.g. homogénea

$$x_p(t) = At + B \Rightarrow A + At + B = t/\pi \Rightarrow A = \frac{1}{\pi}, B = -\frac{1}{\pi}$$

$$x(t) = C e^{-t} + \frac{t}{\pi} - \frac{1}{\pi} \quad \text{solución general.}$$

Condición de continuidad en  $t=0 \Rightarrow x(0)=0 \Rightarrow C - \frac{1}{\pi} = 0 \Rightarrow C = \frac{1}{\pi}$

$| x_{II}(t) = \frac{1}{\pi} \left( e^{-t} + t - 1 \right) \text{ si } 0 \leq t \leq \pi |$

(III)  $x(t) = x_h(t) + x_p(t)$ ,  $x_h(t) = C e^{-t}$   $A = -\frac{1}{\pi}$

$$x_p(t) = At + B \Rightarrow A + At + B = 2 - \frac{t}{\pi} \Rightarrow B = 2 + \frac{1}{\pi}$$

$$x_{III}(t) = C e^{-t} - \frac{t}{\pi} + 2 + \frac{1}{\pi} \quad \text{Condición de continuidad en } t=\pi$$

$$x_{III}(\pi) = x_{II}(\pi) \Rightarrow \frac{1}{\pi} \left( e^{-\pi} + \pi - 1 \right) = C e^{-\pi} + 1 + \frac{1}{\pi} \Rightarrow C = \frac{1}{\pi} (1 - 2e^{\pi})$$

$| x_{III}(t) = \frac{1}{\pi} (1 - 2e^{\pi}) e^{-t} - \frac{t}{\pi} + 2 + \frac{1}{\pi} \quad \text{si } \pi < t \leq 2\pi |$

(IV)  $X_{\frac{V}{II}}(t) = X_h(t) = Ce^{-t}$ . Condición de continuidad en  $t=2\pi$ : L2

$$X_{\frac{V}{III}}(2\pi) = X_{\frac{V}{II}}(2\pi) \Rightarrow \frac{1}{\pi} (1 - 2e^{\pi}) e^{-2\pi} + \frac{1}{\pi} = Ce^{-2\pi} \Rightarrow C = \frac{1}{\pi} (1 - 2e^{\pi} + e^{2\pi})$$

$$\boxed{X_{\frac{V}{IV}}(t) = \frac{1}{\pi} (1 - 2e^{\pi} + e^{2\pi}) \cdot e^{-t} \quad \text{si } t > 2\pi}$$

b)  $\dot{x}(t) + x(t) = f(t) \Rightarrow \hat{x}(\omega) (i\omega + 1) = \hat{f}(\omega) \Rightarrow \hat{x}(\omega) = \frac{\hat{f}(\omega)}{i\omega + 1}$

$$\begin{aligned} \hat{f}(\omega) &= \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_0^{\pi} \frac{t}{\pi} e^{-i\omega t} dt + \int_{\pi}^{2\pi} (2 - \frac{t}{\pi}) e^{-i\omega t} dt \stackrel{\text{partes}}{=} \\ &= \frac{-1 + 2e^{-i\pi\omega} - e^{-2i\pi\omega}}{\pi\omega^2} \end{aligned}$$

$$\hat{x}(\omega) = \frac{1}{\pi} \frac{-1 + 2e^{-i\pi\omega} - e^{-2i\pi\omega}}{(i\omega + 1)\omega^2} = \frac{-1}{\pi} \left( \frac{1}{(i\omega + 1)\omega^2} + \frac{-2e^{-i\pi\omega}}{(i\omega + 1)\omega^2} + \frac{e^{-2i\pi\omega}}{(i\omega + 1)\omega^2} \right)$$

$$\mathcal{F}^{-1} \left( \frac{1}{(i\omega + 1)\omega^2} \right) \stackrel{\text{fracciones simples}}{=} \mathcal{F}^{-1} \left( \frac{-1}{(i\omega + 1)} + \frac{-i}{\omega} + \frac{1}{\omega^2} \right) =$$

$$= -e^{-t} u_s(t) + v_s(t) - t v_s(t) \quad \text{con } v_s(t) = u_s(t) - \frac{1}{2} \quad \begin{matrix} \text{salto de} \\ \text{media nula} \end{matrix}$$

Utilizando la propiedad de desplazamiento  $\mathcal{F}^{-1}(\hat{f}(\omega) e^{\pm i\omega t_0}) = f(t \pm t_0)$ :

$$\boxed{X(t) = \mathcal{F}^{-1}(\hat{x}(\omega)) = -\frac{1}{\pi} \left[ -e^{-t} u_s(t) + v_s(t) - t v_s(t) + \right.} \\ \left. + 2e^{-(t-\pi)} u_s(t-\pi) - 2v_s(t-\pi) + 2(t-\pi)v_s(t-\pi) \right. \\ \left. - e^{-(t-2\pi)} u_s(t-2\pi) + v_s(t-2\pi) - (t-2\pi)v_s(t-2\pi) \right]$$

2) Ecación diferencial:  $L\ddot{I} + R\dot{I} + \frac{1}{C}I = E(t)$

3

a)  $E(t) = \sum_{n=-\infty}^{\infty} C_n e^{i\omega_n t}$  donde  $\omega_n = \frac{2\pi}{T} n = n$

$$C_n = \frac{1}{2\pi} \int_0^{2\pi} f(t) e^{-i\omega_n t} dt \stackrel{\text{ejercicio 1}}{=} \frac{-1 + 2e^{-i\pi n} - e^{-2i\pi n}}{2\pi^2 \cdot n^2} = \frac{(-1)^n - 1}{\pi^2 n^2}$$

$$C_0 = \frac{1}{2\pi} \int_0^{2\pi} f(t) dt = \frac{1}{2\pi} \left( \frac{2\pi \cdot 1}{2} \right) = \frac{1}{2}$$

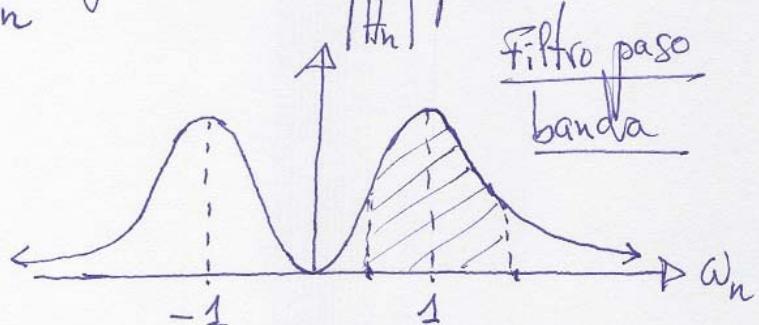
Así,  $C_n = 0$  si  $n$  es par  $\checkmark$  y  $C_n = \frac{-2}{\pi^2 n^2}$  si  $n$  es impar

b)  $\ddot{I} + \dot{I} + I = E \Rightarrow \widehat{I(\omega_n)} (-\omega_n^2 + i\omega_n + 1) = i\omega_n \widehat{E(\omega_n)}$

$$H_n = \frac{D_n}{C_n} = \frac{i\omega_n}{1 - \omega_n^2 + i\omega_n} \quad \text{función de transferencia}$$

$$|H_n| = \left( \frac{\omega_n^2}{(1 - \omega_n^2)^2 + \omega_n^2} \right)^{1/2}$$

$$= \left( \frac{n^2}{(1 - n^2)^2 + n^2} \right)^{1/2}$$



ESPECTRO DE AMPLITUD

$$c) \quad \tilde{E}(t) = \sum_{n=-1}^{n=1} c_n e^{-i\omega_n t} = \frac{1}{2} - \frac{4}{\pi^2} \cos(t)$$

$$L=1 \Rightarrow \ddot{I} + R\dot{I} + \frac{1}{C}I = \tilde{E}(t) \quad \text{Existe resonancia con } n=1 \text{ si } R=0$$

$$\text{y } \frac{1}{C} = \omega_1^2 = 1^2 \Rightarrow C = \frac{1}{1}$$

Así, la ecuación diferencial a resolver es:

$$\ddot{I} + I = \frac{4}{\pi^2} \sin(t) \quad \text{con condiciones iniciales } I(0) = \dot{I}(0) = 0$$

Solución general de la homogénea  $I_h(t) = G \cos(t) + S \sin(t)$

Solución particular  $I_p(t) = At \sin t + Bt \cos t$

que introducida en la ecuación diferencial nos da:  $\begin{cases} A=0 \\ B=-2/\pi^2 \end{cases}$

Así, la solución general es  $I(t) = G \cos(t) + S \sin(t) - \frac{2}{\pi^2} t \cos(t)$

$$\dot{I}(0) = I(0) = 0 \Rightarrow G=0, S=\frac{2}{\pi^2} \Rightarrow$$

$$I(t) = \frac{2}{\pi^2} \left( S \sin(t) - t \cos(t) \right)$$

$$3) a) \boxed{\theta = \frac{\pi}{6}} \quad X_K - X_{K-1} + X_{K-2} = U_K, \quad X_{-1} = X_{-2} = 0$$

$$\text{Ecación Característica } \lambda^2 - \lambda + 1 = 0 \Rightarrow \lambda = \frac{1 \pm \sqrt{3}}{2} = e^{\pm i \frac{\pi}{3}}$$

$$\text{Solución general de la ecación homogénea } X_h[K] = C_1 \cos\left(k \frac{\pi}{3}\right) + C_2 \sin\left(k \frac{\pi}{3}\right)$$

Solución particular  $X_p[K] = A U_K \Rightarrow A(U_K - U_{K-1} + U_{K-2}) = U_K$ . Tomando  $K \geq 2$  (instante a partir del cual ninguno de los escalones se anula)  $\Rightarrow A=1$

$$\text{Solución general: } X_K = C_1 \cos\left(k \frac{\pi}{3}\right) + C_2 \sin\left(k \frac{\pi}{3}\right) + U_K \quad \boxed{K \geq 2} \quad \begin{matrix} \text{Valida para} \\ K \geq 2 \end{matrix}$$

Hay que imponer las condiciones iniciales en  $K=0$  y  $K=1$  luego.

Ecación en diferencias

solución general

$$X_0 = X_{-1}^0 - X_{-2}^0 + U_0 = 1 \stackrel{X_{-1}^0 = X_{-2}^0}{=} C_1 \cos(0 \cdot \frac{\pi}{3}) + C_2 \sin(0 \cdot \frac{\pi}{3}) + 1 \quad \Rightarrow \quad C_1 = 0$$

$$X_1 = X_0 - X_{-1}^0 + U_1 = 2 = C_1 \cos\left(\frac{\pi}{3}\right) + C_2 \sin\left(\frac{\pi}{3}\right) + 1 \quad \Rightarrow \quad C_2 = \frac{2}{\sqrt{3}}$$

$$\text{Solución:} \quad \boxed{\dots - X_K = \left[ \frac{2}{\sqrt{3}} \sin\left(k \frac{\pi}{3}\right) + 1 \right] U_K}$$

$$b) \boxed{\theta = \frac{\pi}{2}} \quad X_K - 2X_{K-1} + X_{K-2} = U_K, \quad X_{-1} = X_{-2} = 0$$

$$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = 1 \text{ doble} \Rightarrow X_h[K] = C_1 + K C_2 \quad \underline{\text{resonancia}}$$

$$X_p[K] = A K^2 U_K \Rightarrow A((K)^2 U_K - 2(K-1)^2 U_{K-1} + (K-2)^2 U_{K-2}) = U_K \stackrel{K \geq 2}{\Rightarrow} A = \frac{1}{2}$$

$$X_K = C_1 + K C_2 + \frac{1}{2} K^2 U_K \quad \begin{matrix} \text{Valida para} \\ K \geq 2 \end{matrix}$$

$$\begin{aligned} X_0 &= 2X_{-1}^0 - X_{-2}^0 + U_0 = 1 = C_1 \\ X_1 &= 2X_0 - X_{-1} + U_1 = 3 = C_1 + C_2 + \frac{1}{2} \\ C_2 &= \frac{3}{2} \end{aligned} \quad \Rightarrow \quad \boxed{X_K = \left(1 + \frac{3}{2}K + \frac{1}{2}K^2\right) U_K}$$