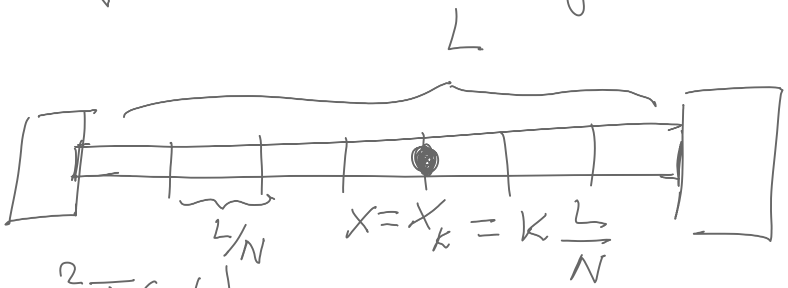
Propagación del Calor.

$$\frac{\partial T(\vec{r},t)}{\partial t} = \frac{\kappa}{\rho c}\left(\frac{\partial^2 T(\vec{r},t)}{\partial x^2} + \frac{\partial^2 T(\vec{r},t)}{\partial y^2} + \frac{\partial^2 T(\vec{r},t)}{\partial z^2}\right)$$

A | VÁSTAGO

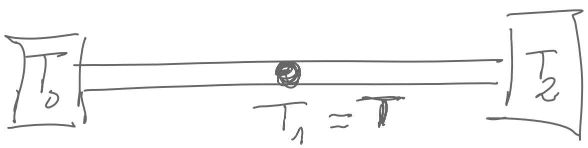


N trozos

$$\frac{\partial T(x,t)}{\partial t} = \frac{\kappa}{\lambda c}\frac{\partial^2 T(x,t)}{\partial x^2} \Longrightarrow T(x_k,t) = T_k(t)$$

$$\dot{T}_k(t) = \frac{\kappa}{\lambda c}\,\frac{T_{k+1}(t) - 2T_k(t) + T_{k-1}(t)}{(L/N)^2}$$

$\boxed{N=2}$



$T_0 = 0°$ $\qquad \kappa = \lambda = c = L = 1$

$T_2 = 100°$ $\qquad T_1(0) = 25°$ Condición inicial

$$\dot{T}_1(t) = \dot{T}(t) = \frac{\kappa}{\lambda c (L/2)^2} \left(T_2 - 2T + T_0 \right)$$

$$\dot{T}(t) = 4 \left(100 - 2T + 0 \right) \Rightarrow \boxed{\dot{T} + 8T = 400}$$

Homogenea $\dot{T} + 8T = 0$, Ensayo $T(t) = e^{\lambda t} \Rightarrow \lambda + 8 = 0 \Rightarrow \lambda = -8$

$T_h(t) = C e^{-8t}$ solución general de la ec. homogenea

No homog. $T_p(t) = A \leftarrow$ Ensayo esta solución particular.

$\cancel{\dot{T}_p}^{\,0} + 8T_p = 400 \Rightarrow 8A = 400 \Rightarrow A = \frac{400}{8} = 50$

Solución general : $T(t) = C e^{-8t} + 50$

Condición inicial $T(0) = 25 = C + 50 \Rightarrow C = -25$

$$\boxed{T(t) = -25\, e^{-8t} + 50} \xrightarrow{t \to \infty} T(\infty) = 50$$

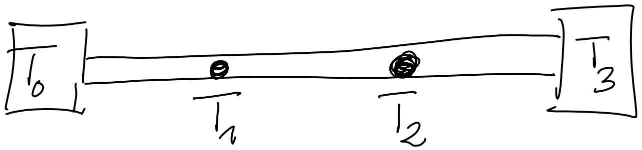
$$\dot{T}(t) = \frac{k}{\lambda c (L/2)^2}\left(T_2 - 2T + T_0\right)$$

En el estado estacionario $\dot{T}(t) = 0 \Rightarrow T_2 - 2T + T_0 = 0$

$$\Rightarrow \boxed{T = \frac{T_0 + T_2}{2}}$$

media aritmética de las temperaturas adyacentes

$\boxed{N = 3}$ $L = 3$, $k = \lambda = c = 1$



$T_0 = 0^\circ$

$T_3 = 100\, e^{-at}$

$T_1(0) = T_2(0) = 0$ condic. inicial

$$\dot{T}_k(t) = \frac{k}{\lambda c}\,\frac{T_{k+1}(t) - 2T_k(t) + T_{k-1}(t)}{(L/N)^2}$$

$$\begin{cases} \dot{T}_1 = T_2 - 2T_1 + T_0 = T_2 - 2T_1 \\ \dot{T}_2 = T_3 - 2T_2 + T_1 = 100 e^{-2t} - 2T_2 + T_1 \end{cases}$$

$$\left.\begin{array}{l}\dot{T}_1 + 2T_1 - T_2 = 0 \\ \dot{T}_2 + 2T_2 - T_1 = 100\, e^{-at}\end{array}\right\} \Rightarrow \begin{pmatrix} D+2 & -1 \\ -1 & D+2 \end{pmatrix}\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 100\, e^{-at} \end{pmatrix}$$

$$D = \frac{d}{dt}$$

$$\underbrace{\begin{vmatrix} D+2 & -1 \\ -1 & D+2 \end{vmatrix}}_{(D+2)^2 - 1} T_1 = \underbrace{\begin{vmatrix} 0 & -1 \\ 100\, e^{-at} & D+2 \end{vmatrix}}_{100\, e^{-at}} \Rightarrow \boxed{\ddot{T}_1 + 4\dot{T}_1 + 3T_1 = 100\, e^{-at}}$$

<u>Homogenea</u> $\ddot{T}_1 + 4T_1 + 3T_1 = 0$, Ensayo $T_1(t) = e^{\lambda t}$

$$(\lambda+2)^2 - 1 = 0 \Longrightarrow (\lambda_\pm + 2) = \pm 1 \Rightarrow \lambda_\pm = -1, -3$$

Solución general de la homogenea $T_1(t) = c_1 e^{-t} + c_2 e^{-3t}$

No homog.

$\boxed{1) \quad a \neq 1, 3}$ $\Rightarrow$ No resonancia $\xrightarrow{\text{Ensayo}}$ $T_{1p}(t) = A e^{-at}$

$$\ddot{T}_{1p} + 4\dot{T}_{1p} + 3T_{1p} = 100 e^{-at} \Rightarrow A(a^2 - 4a + 3) = 100$$

$$\Rightarrow A = 100/(a^2-4a+3)$$

Solución general $T_1(t) = C_1 e^{-t} + C_2 e^{-3t} + \frac{100}{a^2-4a+3} e^{-at}$

$$T_2 = \dot{T}_1 + 2T_1 = -C_1 e^{-t} - 3C_2 e^{-3t} - a\frac{100}{a^2-4a+3} e^{-at} +$$

$$2C_1 e^{-t} + 2C_2 e^{-3t} + 2\frac{100}{a^2-4a+3} e^{-at}$$

$$= C_1 e^{-t} - C_2 e^{-3t} + (2-a)\frac{100}{a^2-4a+3} e^{-at}$$

Condiciones iniciales

$$T_1(0) = 0 = C_1 + C_2 + \frac{100}{a^2-4a+3}$$
$$T_2(0) = 0 = C_1 - C_2 + (2-a)\frac{100}{a^2-4a+3}$$
$$\left.\right\} \quad C_1 = \dots \qquad C_2 = \dots$$

$\boxed{2) \quad a=1}$ $\quad \ddot{T}_1 + 4\dot{T}_1 + 3T_1 = 100e^{-t}$

"resonancia"

$$T_{1h}(t) = C_1 e^{-t} + C_2 e^{-3t}, \qquad T_{1p} = Ae^{-t}t$$

$$\dot{T}_{1p} = Ae^{-t}(1-t), \qquad \ddot{T}_{1p} = -Ae^{-t}(1-t) - Ae^{-t} = Ae^{-t}(-2+t)$$

$$\ddot{T}_{1p} + 4\dot{T}_{1p} + 3T_{1p} = Ae^{-t}\Big(\underbrace{(-2+t) + 4(1-t) + 3t}_{2}\Big) = 2Ae^{-t} = 100e^{-t}$$
$$A = 50$$

$$T_1(t) = C_1e^{-t} + C_2e^{-3t} + 50te^{-t}$$
$$T_2(t) = \dot{T}_1 + 2T_1 = C_1e^{-t} - C_2e^{-3t} + 50e^{-t} - 50te^{-t} + 2\cdot 50te^{-t}$$
$$= C_1e^{-t} - C_2e^{-3t} + 50(t+1)e^{-t}$$

$$\left.\begin{aligned} T_1(0) = 0 &= C_1 + C_2 \\ T_2(0) = 0 &= C_1 - C_2 + 50 \\ \hline 2C_1 + 50 &= 0 \end{aligned}\right\} \qquad \begin{aligned} C_1 &= -25 \\ C_2 &= 25 \end{aligned}$$

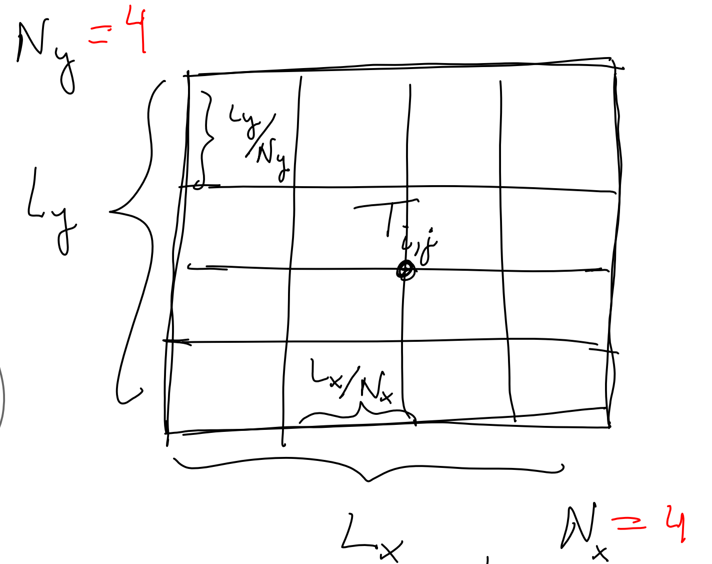
$$\boxed{\begin{aligned} T_1(t) &= -25e^{-t} + 25e^{-3t} + 50te^{-t} \\ T_2(t) &= -25e^{-t} - 25e^{-3t} + 50(t+1)e^{-t} \end{aligned}} \quad \begin{aligned} &\xrightarrow{t\to\infty} T_1(\infty) = 0 \\ &\xrightarrow{t\to\infty} T_2(\infty) = 0 \end{aligned}$$

$$\left(\dot{T}_1(t) = 25e^{-t} - 75e^{-3t} + 50e^{-t} - 50te^{-t} = 0\right) / e^{-t}$$

$$25 - 75e^{-2t} + 50 - 50t = 0 \rightarrow \boxed{75e^{-2t_0} + 50t_0 = 75}$$

t_0 es el instante de tiempo en que $T_1(t)$ es máxima.

B | PLACA



$$\frac{\partial T(\vec{r},t)}{\partial t} = \frac{\kappa}{\rho c}\left(\frac{\partial^2 T(\vec{r},t)}{\partial x^2} + \frac{\partial^2 T(\vec{r},t)}{\partial y^2}\right)$$

$$T(x_i, y_j, t) = T_{ij}(t) \qquad \Big| \qquad x_i = i\,\frac{L}{N} \qquad y_j = j\,\frac{L}{N}$$

$$L_x = L_y = L\,,\quad N_x = N_y = N$$

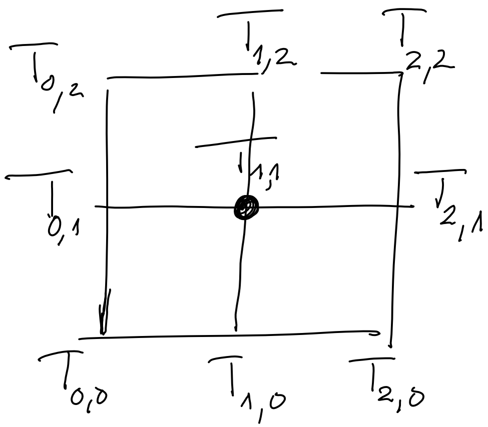
$$\dot{T}_{ij}(t) = \frac{\kappa}{\rho c\left(\frac{L}{N}\right)^2}\left(T_{i-1,j}(t) + T_{i+1,j}(t) + T_{i,j-1}(t) + T_{i,j+1}(t) - 4\,T_{ij}(t)\right)$$

he utilizado que

$$\begin{cases} \dfrac{\partial^2 T(x_i, y_j, t)}{\partial x^2} \simeq \left(T_{i+1,j} - 2\,T_{ij} + T_{i-1,j}\right)\Big/\left(\frac{L}{N}\right)^2 \\[2ex] \dfrac{\partial^2 T(x_i, y_j, t)}{\partial y^2} \simeq \left(T_{i,j+1} - 2\,T_{ij} + T_{i,j-1}\right)\Big/\left(\frac{L}{N}\right)^2 \end{cases}$$

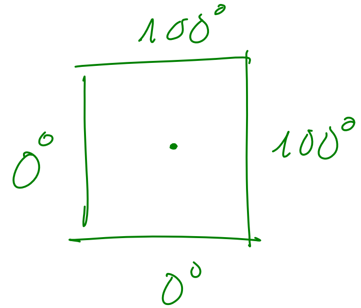
Si L/N es pequeño ...

$$\boxed{N=2}$$



$$\dot{T}_{11} = \dot{T} = \frac{k}{\rho c \left(\frac{L}{N}\right)^2}\left(T_{01} + T_{21} + T_{1,0} + T_{12} - 4T\right)$$

Cond. de Contorno



$\rho = c = k = 1, \quad L = 2$

Condición inicial $T(0) = 0$

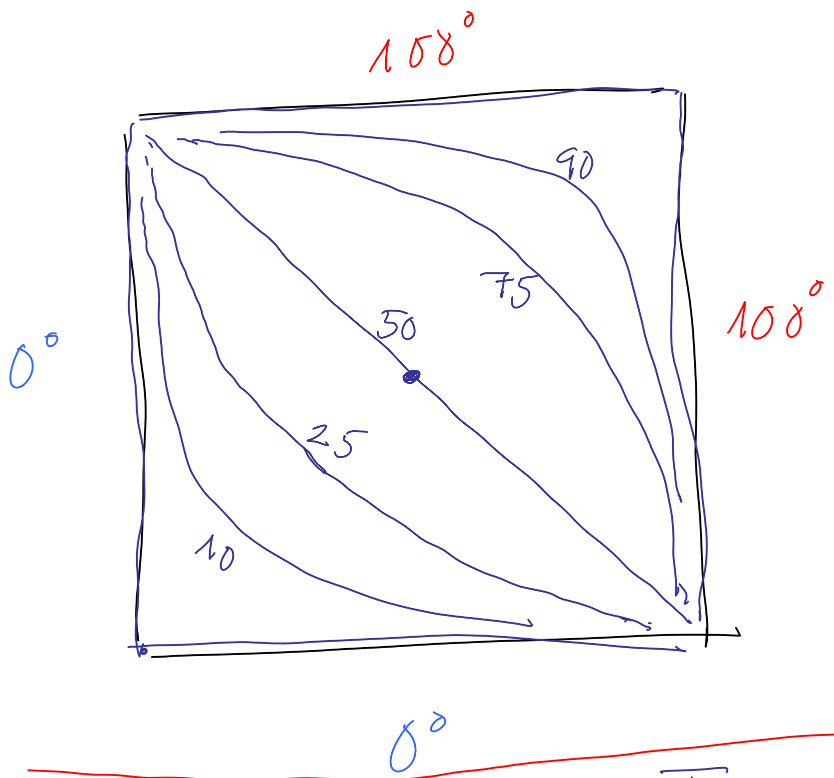
$$\dot{T} = (0 + 100 + 0 + 100 - 4T) \Rightarrow \dot{T} + 4T = 200$$

$$T_h(t) = C e^{-4t}, \quad T_p(t) = A \Rightarrow A = \frac{200}{4} = 50$$

$$T(t) = Ce^{-4t} + 50, \quad T(0) = 0 = C + 50 \Rightarrow C = -50$$

$$\boxed{T(t) = -50e^{-4t} + 50} \xrightarrow{t \to \infty} T(\infty) = 50$$

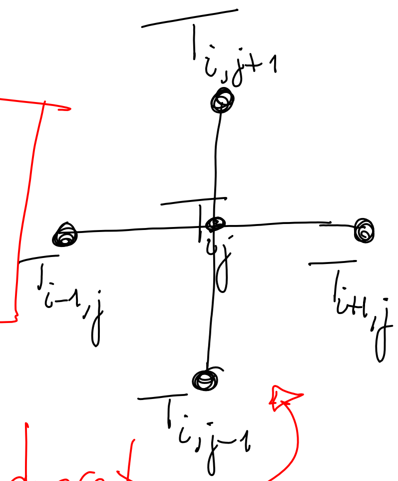
Estado estacionario



Isotermas en el estado estacionario

Estado estacionario

$$\dot{T}_{i,j} = 0 \Rightarrow \boxed{T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}}$$



En el estado estacionario, la temperatura en cada punto es la media aritmética de la Tª en los puntos adyacentes