Examen de noviembre 2004

Título de la nota — 24/10/2008

Represento gráficamente $f(x) = (x^2+x-2)e^{-2x}$

$$f'(x) = (2x+1)e^{-2x} + (x^2+x-2)(-2)e^{-2x} =$$

$$= e^{-2x}(\cancel{2x}+1-2x^2-\cancel{2x}+4) = e^{-2x}(-2x^2+5) = 0$$

$$\Rightarrow -2x^2+5=0 \Rightarrow x^2 = \frac{5}{2} \Rightarrow x_\pm = \pm\sqrt{\frac{5}{2}}$$

$$f''(x) = -2e^{-2x}(-2x^2+5) + e^{-2x}(-4x) = e^{-2x}(4x^2-10-4x) = 0$$

$$\frac{4x^2-10-4x}{2} = 0 \Rightarrow 2x^2-2x-5=0 \Rightarrow x_\pm = \frac{2\pm\sqrt{4+40}}{4}$$

$$x_{i\pm} = \frac{1}{2}(1\pm\sqrt{11})$$

$$f''\left(\pm\sqrt{\tfrac{5}{2}}\right) = \underbrace{2e^{\mp 2\sqrt{\frac{5}{2}}}}_{>0}\underbrace{\left(2\left(\pm\sqrt{\tfrac{5}{2}}\right)^2 \mp 2\sqrt{\tfrac{5}{2}} - 5\right)}$$

$+ \rightarrow \cancel{2}\frac{5}{\cancel{2}} - 2\sqrt{\frac{5}{2}} - \cancel{5} < 0$

$- \rightarrow \cancel{2}\frac{5}{\cancel{2}} + 2\sqrt{\frac{5}{2}} - \cancel{5} > 0$

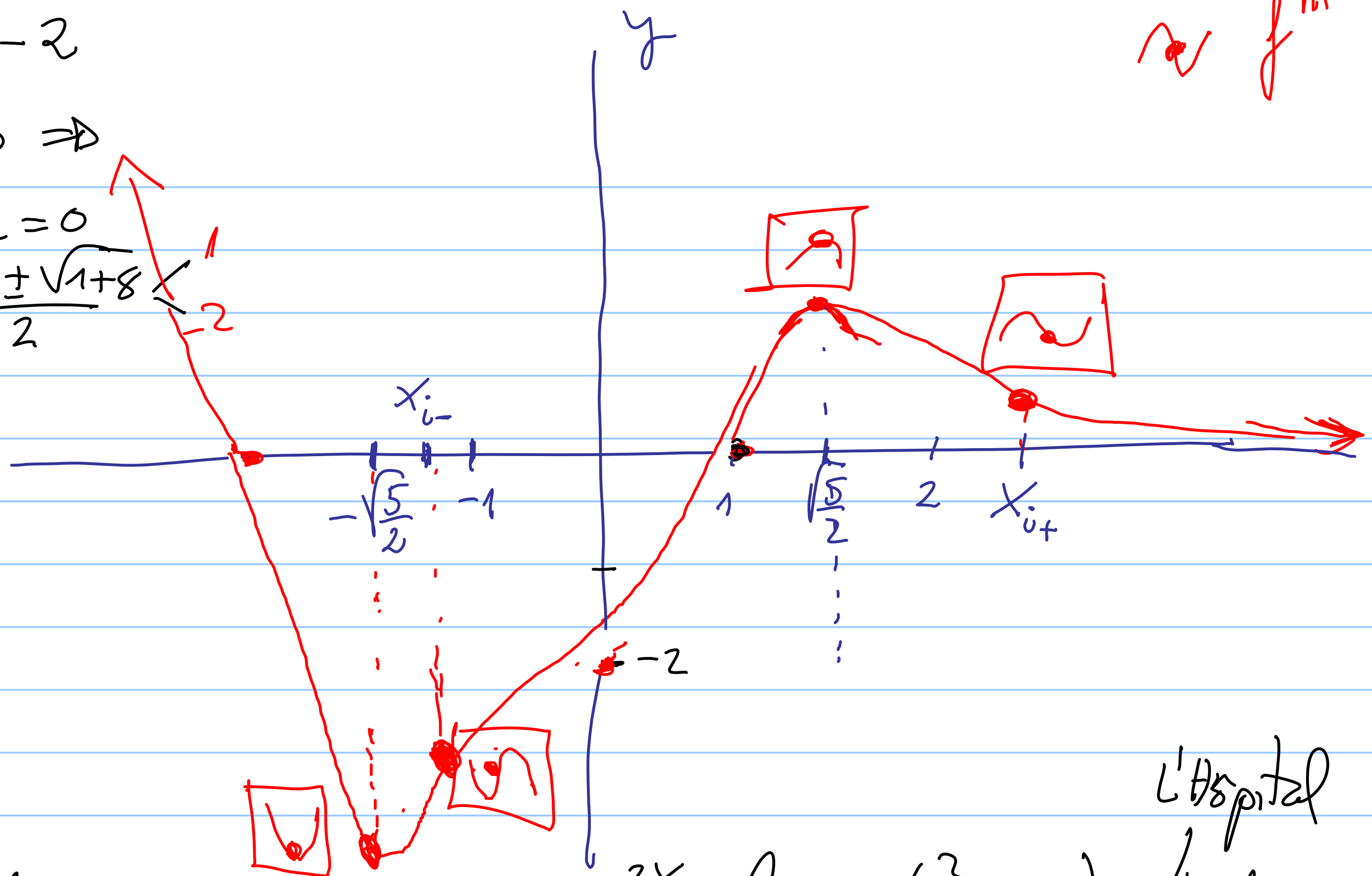
$f(0) = -2$

$f(x) = 0 \Rightarrow$

$x^2+x-2=0$

$x = \frac{-1 \pm \sqrt{1+8}}{2} \begin{matrix} 1 \\ -2 \end{matrix}$

$\sim f'''$



$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \underbrace{(x^2+x-2)}_{\infty}\,\underbrace{e^{-2x}}_{0} = \lim_{x\to\infty} \frac{(x^2+x-2)}{e^{2x}} \overset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{2x+1}{2e^{2x}} =$$

$$\lim_{x\to\infty} \frac{2}{4e^{2x}} = \boxed{0}$$

$$\lim_{x\to-\infty} \underbrace{(x^2+x-2)}_{\infty} \cdot \underbrace{e^{-2x}}_{\infty} = \infty$$