SOLUCIONES RELACIÓN DE PROBLEMAS. ARQUITECT. TECNICO 2003-2004

1a) $\lim_{x\to\infty} f(x) = \infty \cdot \text{signo}\left(\frac{a_n}{b_m}\right)$; $\lim_{x\to-\infty} f(x) = \infty \cdot \text{signo}\left(\frac{a_n}{b_n}\right)\cdot(-1)^{n+m}$; $\lim_{x\to 0} f(x) = \frac{a_0}{b_0}$

1b) $\lim_{x\to\infty} f(x) = 0 = \lim_{x\to-\infty} f(x)$; $\lim_{x\to 0} f(x) = \frac{a_0}{b_0}$

1.c) $\lim_{x\to\infty} f(x) = \frac{a_n}{b_n} = \lim_{x\to-\infty} f(x)$; $\lim_{x\to 0} f(x) = \frac{a_0}{b_0}$

2.a) $\lim_{x\to\pm\infty,\ 0} |x|^{-1/2}\left(\frac{1}{2}\right)^x = \begin{cases} 0 \\ \infty \\ \infty \end{cases}$, 2.b) $\lim_{x\to\pm\infty,\ 0} |x|^{-3/2}\left(\frac{1}{3}\right)^x = \begin{cases} 0 \\ \infty \\ \infty \end{cases}$, 2.c) $\lim_{x\to\pm\infty,\ 0} |x|^{1/2}\left(\frac{1}{4}\right)^x = \begin{cases} 0 \\ \infty \\ 0 \end{cases}$

2.d) $\lim_{x\to\pm\infty,\ 0} |x|^{2}\left(\frac{1}{5}\right)^x = \begin{cases} 0 \\ \infty \\ 0 \end{cases}$, 2.e) $\lim_{x\to\pm\infty,\ 0} |x|^{-1/2}\, 2^x = \begin{cases} \infty \\ 0 \\ \infty \end{cases}$, 2.f) $\lim_{x\to +\infty,\ -\infty,\ 0} |x|^{-3/2}\, 3^x = \begin{cases} \infty \\ 0 \\ \infty \end{cases}$

2.g) $\lim_{x\to\pm\infty,\ 0} |x|^{1/2}\, 4^x = \begin{cases} \infty \\ 0 \\ 0 \end{cases}$ 2.h) $\lim_{x\to\pm\infty,\ 0} |x|^{2}\, 5^x = \begin{cases} \infty \\ 0 \\ 0 \end{cases}$

3.a) $\lim_{x\to\infty,\ 0} |x|^{-1/2}\log_2|x| = \begin{cases} 0 \\ -\infty \end{cases}$ 3.b) $\lim_{x\to\infty,\ 0} |x|^{-3/2}\log_3|x| = \begin{cases} 0 \\ -\infty \end{cases}$

3.c) $\lim_{x\to\infty,\ 0} |x|^{1/2}\log_e|x| = \begin{cases} \infty \\ 0 \end{cases}$ 3.d) $\lim_{x\to\infty,\ 0} |x|^{3/2}\log_{10}|x| = \begin{cases} \infty \\ 0 \end{cases}$

4.a) $\lim_{x\to\pm\infty,\ 0} \left(\frac{1}{2}\right)^x \log_2|x| = \begin{cases} 0 \\ \infty \\ -\infty \end{cases}$ 4.b) $\lim_{x\to\pm\infty,\ 0} 2^x \log_e|x| = \begin{cases} \infty \\ 0 \\ -\infty \end{cases}$

5.a) $\lim_{x\to 0} \frac{\operatorname{sen}(ax)}{bx} = \frac{a}{b}$, 5.b) $\lim_{x\to 0} \frac{\operatorname{sen}(\alpha x)}{\tan(\beta x)} = \frac{\alpha}{\beta}$, 5.c) $\lim_{x\to\infty} x \operatorname{sen} x$ no existe

5.d) $\lim_{x\to\infty} \frac{\operatorname{sen} x}{x} = 0$

6.a) $\left(b^x \log_a|1/x|\right)' = -\frac{b^x}{x\ln(a)} + \frac{b^x \ln(b)\ln\left(\frac{1}{x}\right)}{\ln(a)}$; 6.b) $\left(a^{-\frac{1}{x^2}}\right)' = \frac{2\, a^{-\frac{1}{x^2}}\cdot \ln(a)}{x^3}$

6.c) $\left(\operatorname{sen}^3(a \arccos(x))\right)' = -\frac{3a \cos(a \arccos(x)) \operatorname{sen}^2(a\cdot\arccos(x))}{\sqrt{1-x^2}}$

6.d) $\left(\arctan(a \tan(ax))\right)' = \frac{a^2 \sec^2(ax)}{1+a^2\tan^2(ax)}$

6.e) $\left(\dfrac{\sqrt{1+\ln\left(\tan\left(\frac{x}{2}\right)\right)}}{e^{-3\sin(5x)}}\right)' = \dfrac{e^{3\sin(5x)}\left[60\cos(5x)\left(1+\ln\left(\tan\left(\frac{x}{2}\right)\right)\right)+\operatorname{cosec}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\right]}{4\sqrt{1+\ln\left(\tan\left(\frac{x}{2}\right)\right)}}$

6.f) $\left(x^{1/x}\right)' = x^{1/x}\cdot\left(\dfrac{1}{x^2}-\dfrac{\ln(x)}{x^2}\right)$

7.a) $\ln x = 0+(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3+\frac{1}{4!}\frac{6}{\theta^4}(x-1)^4 \quad , \frac{1}{3}<\theta<1$

$$\boxed{\ln\left(\tfrac{1}{3}\right) = -\frac{80}{81} \pm 4}$$

$\left|\frac{6}{\theta^4}\right| \leq \left|\frac{6}{(1/3)^4}\right| = 486$

7.b) $e^x = 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{1}{4!}e^{\theta}(x-0)^4 \qquad 0<\theta<\frac{1}{5}$

$$\boxed{e^{1/5} = \frac{458}{375} \pm \frac{2}{15\,000}}$$

$|e^{\theta}| \leq e^{1/5} < 2$

7.c) $\sin x = \frac{1}{2}+\frac{1}{2}\sqrt{3}\left(x-\frac{\pi}{6}\right)-\frac{1}{4}\left(x-\frac{\pi}{6}\right)^2-\frac{1}{4\sqrt{3}}\left(x-\frac{\pi}{6}\right)^3+\frac{\sin(\theta)}{4!}\left(x-\frac{\pi}{6}\right)^4$

$$\boxed{\begin{array}{l}\sin\left(\frac{\pi}{6}+\frac{\pi}{10}\right) = \dfrac{6000+600\sqrt{3}\pi-30\pi^2-\sqrt{3}\pi^3}{1200} \pm \dfrac{\pi^4}{240.000} \\ \qquad \approx 0'7429 \pm 0'0004\end{array}}$$

$\frac{\pi}{6}<\theta<\frac{\pi}{6}+\frac{\pi}{10}$

$|\sin\theta| \leq \sin\left(\frac{\pi}{2}\right) = 1$

7d) $\sqrt{x} = 2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2+\frac{1}{512}(x-4)^3-\frac{1}{4!}\frac{15}{16\theta^{7/2}}(x-4)^4$

$4<\theta<5$

$$\boxed{\sqrt{5} = \frac{1145}{512} \pm \frac{5}{16384}}$$

$\left|\frac{15}{16\theta^{7/2}}\right| \leq \frac{15}{2048}$

7.e) $\cos(x) = \frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)-\frac{1}{2\sqrt{2}}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{6\sqrt{2}}\left(x-\frac{\pi}{4}\right)^3+\frac{\cos\theta}{4!}\left(x-\frac{\pi}{4}\right)^4$

$\frac{\pi}{4}<\theta<\frac{\pi}{4}+\frac{\pi}{15}$

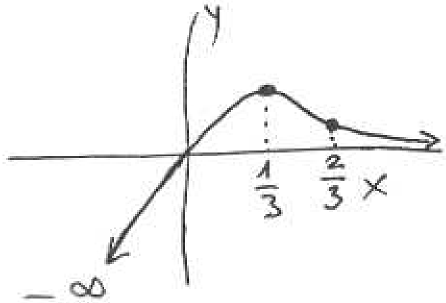
$\cos\left(\frac{\pi}{4}+\frac{\pi}{15}\right) = \dfrac{20250-1350\pi-45\pi^2+\pi^3}{20250\sqrt{2}} \pm \dfrac{\pi^4}{1215000\sqrt{2}}$

$|\cos\theta| \leq \frac{1}{\sqrt{2}}$

$\simeq 0'54459 \pm 0'00006$

8.a) $f(x) = x e^{-3x}$, $f'(x) = 0 \Rightarrow x = \frac{1}{3}$ max

$f''(x) = 0 \Rightarrow x = \frac{2}{3}$ inflex

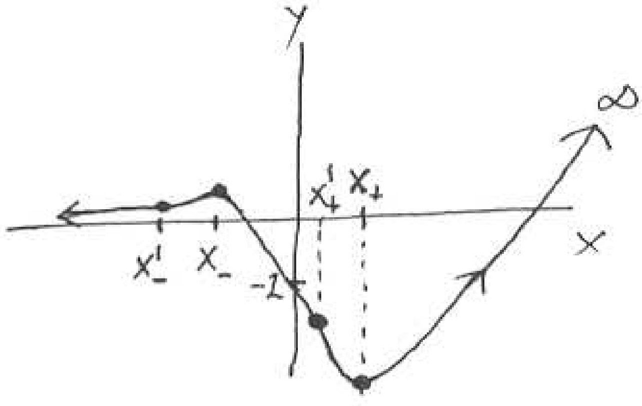


8.b) $f(x) = (x^2 - 2) e^{5x}$

$f'(x) = 0 \Rightarrow \begin{cases} x_+ = \frac{1}{5}(-1-\sqrt{51}) & \text{max} \\ x_- = \frac{1}{5}(-1+\sqrt{51}) & \text{min.} \end{cases}$

$f''(x) = 0 \Rightarrow x'_+ = \frac{2}{5}(-1-\sqrt{13})$ inflex

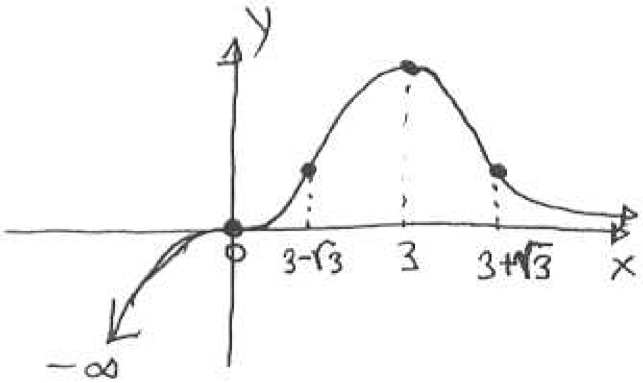
$x'_- = \frac{2}{5}(-1+\sqrt{13})$ inflex



8.c) $f(x) = x^3 e^{-x}$

$f'(x) = 0 \Rightarrow \begin{cases} x = 0 & \text{inflex.} \\ x = 3 & \text{máximo} \end{cases}$

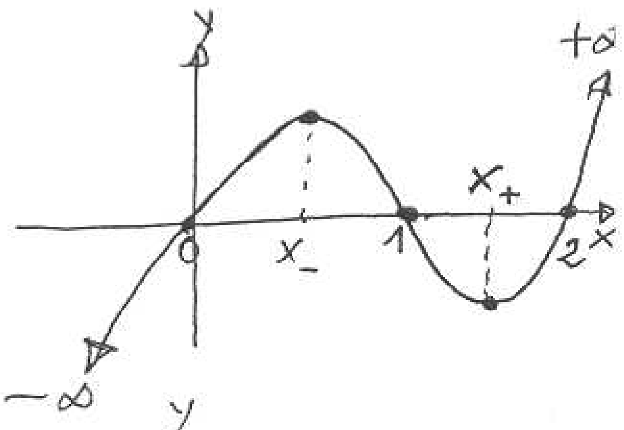
$f''(x) = 0 \Rightarrow \begin{cases} x = 0 \\ x_\pm = 3 \pm \sqrt{3} & \text{inflex.} \end{cases}$



8.d) $f(x) = x(x-1)(x-2)$

$f'(x) = 0 \Rightarrow x_\pm = \frac{1}{3}(3 \pm \sqrt{3})$ min max

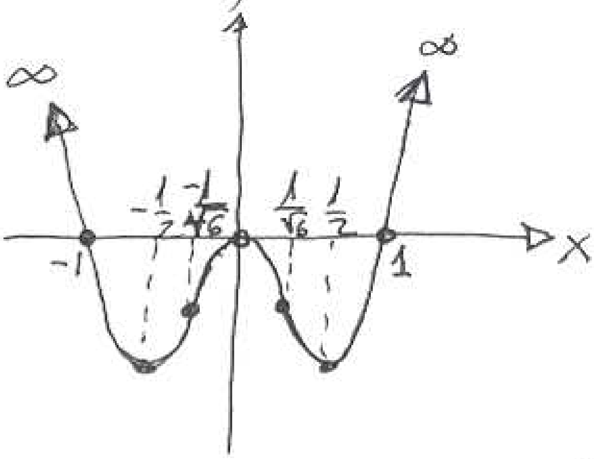
$f''(x) = 0 \Rightarrow x_i = 1$ inflex



8.e) $f(x) = x^2(x^2 - 1)$

$f'(x) = 0 \Rightarrow \begin{cases} x = 0 & \text{max} \\ x = -\frac{1}{2} & \text{min} \\ x = +\frac{1}{2} & \text{min} \end{cases}$

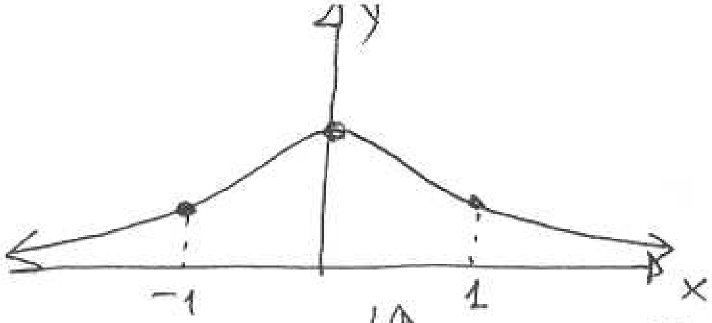
$f''(x) = 0 \Rightarrow x_\pm = \pm \frac{1}{\sqrt{6}}$ inflex



8.f) $f(x) = \dfrac{4}{x^2+3}$

$f'(x) = 0 \Rightarrow x = 0$ max

$f''(x) = 0 \Rightarrow x = \pm 1$ inflex



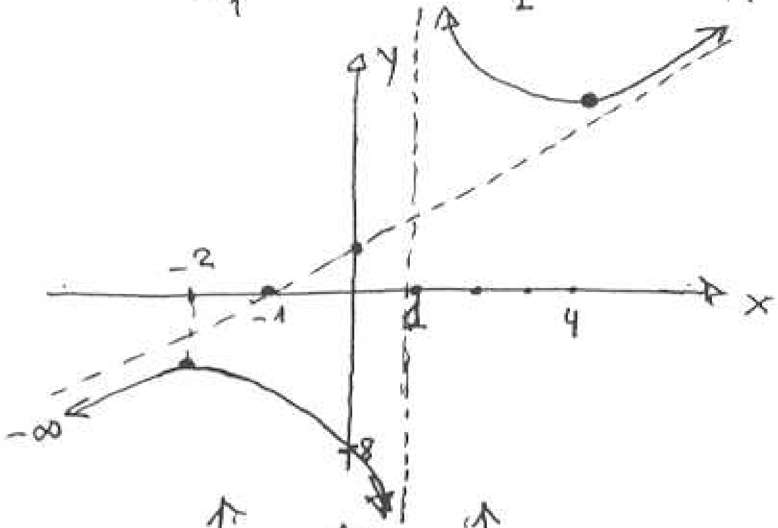
8.g) $f(x) = \dfrac{x^2+8}{x-1}$

$f'(x) = 0 \Rightarrow$ $x = -2$ max
$x = 4$ min

no hay inflex

Asint. Vertical en $x = 1$

" oblicua $y = x+1$

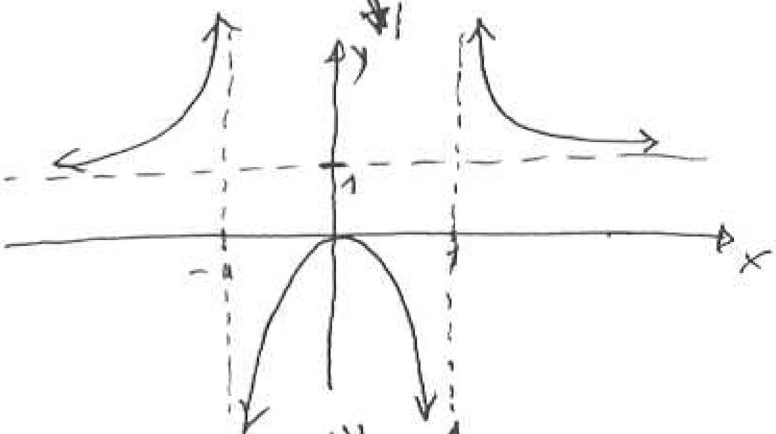


8.h) $f(x) = \dfrac{x^2}{x^2-1}$

$f'(x) = 0 \Rightarrow x = 0$ max

Asint. horizontal $y = 1$

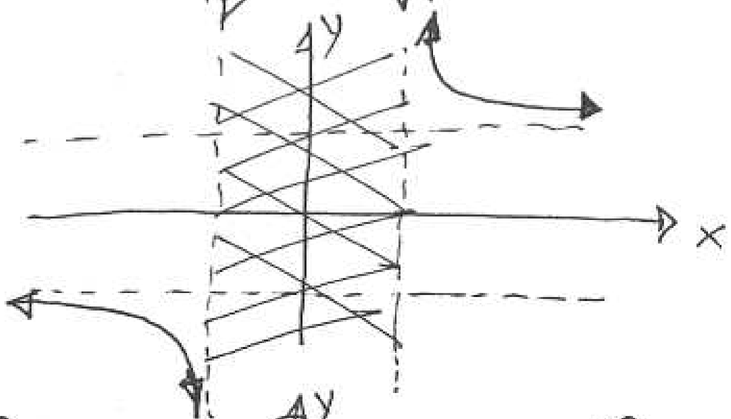
" vertical $x = \pm 1$



8.i) $f(x) = \dfrac{x}{\sqrt{x^2-1}}$

Dominio$(f) = \mathbb{R} -]-1,1[$

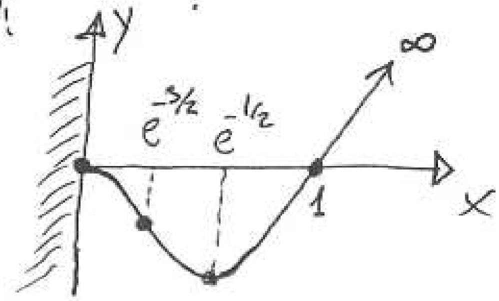
Asintota horizontal $y = \pm 1$



8.j) $f(x) = x^2 \ln(x)$ $\quad \lim_{x \to 0} f(x) = 0$

$f'(x) = 0 \Rightarrow x = e^{-1/2}$ minimo $\quad x = 0$ maximo

$f''(x) = 0 \Rightarrow x = e^{-3/2}$ inflex



8.k) $f(x) = \ln|x^2-5x+6|$

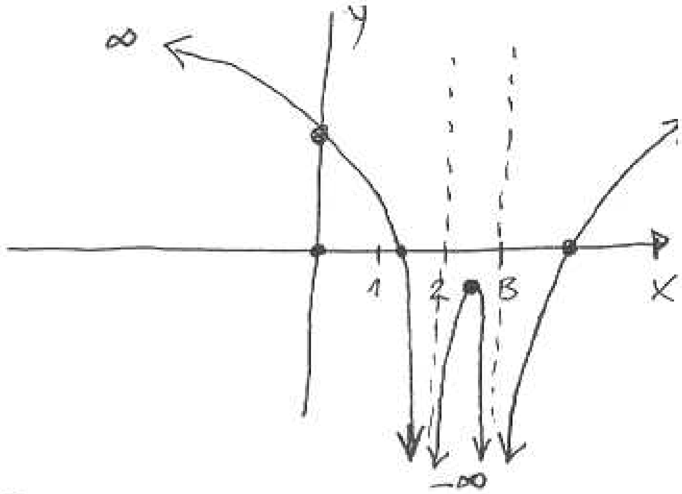
Asintot. verticales:

$x^2-5x+6=0 \Rightarrow x = \frac{5\pm\sqrt{25-24}}{2} = 3, 2$

$f'(x)=0 \Rightarrow x=\frac{5}{2}$ máximo

Ptos de corte $f(x)=0 \Rightarrow x = \frac{5\pm\sqrt{5}}{2}$

$f(0)=\ln(6)$

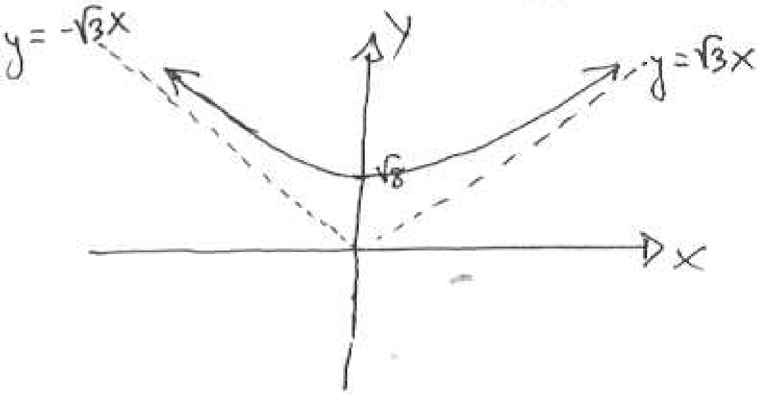


8.l) $f(x)=\sqrt{3x^2+8}$

$f'(x)=0 \Rightarrow x=0$

$f''(x)=0 \Rightarrow$ no hay

Asint. oblicua $y=\pm\sqrt{3}x$



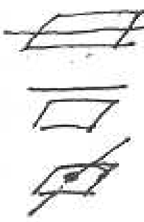
9| 9a) $B_0(H_1)=\left\{\left(0,\frac{2}{\sqrt5},\frac{1}{\sqrt5}\right),\left(\sqrt{\frac{5}{14}},-\frac{3}{70},3\sqrt{2/35}\right)\right\}$

9b) $\overrightarrow{\operatorname{Proy}^{\perp}_{H_1}(\vec p)} = \left(\frac{5}{14},\frac{25}{14},\frac{10}{7}\right)$, 9.c) $\operatorname{Proy}^{H_2}_{H_1}(\vec p) =$ "infinita"

9.d) $\overrightarrow{d^{\perp}(\vec p, H_1)} = \vec p - \operatorname{Proy}^{\perp}_{H_1}(\vec p) = \left(\frac{9}{14},\frac{3}{14},-\frac{3}{7}\right)$, $\|d^{\perp}(\vec p,H_1)\| = \frac{3}{\sqrt{14}}$

9.e) $\widehat{(\vec p, H_1)} = \arccos\dfrac{\vec p\cdot\overrightarrow{\operatorname{Proy}_{H_1}(\vec p)}}{\|\vec p\|\cdot\|\operatorname{Proy}^{\perp}\|} = \arccos\left(\frac{5}{2\sqrt7}\right) \simeq 0'33$ rad.

9.f) $\left.\begin{array}{r} 3x+y-2z=0\\ x-y\quad\;\;=0\\ by-z=-a\end{array}\right\}$
- $b=2,\ a=0$ Comp. indet.
- $b=2,\ a\neq 0$ incomp.
- $b\neq 2$ Compat. determinado.



10| 10a) $H_1^{\perp}=\langle(0,2,1)\rangle$, $H_2^{\perp}=\{x-y+2z=0\}$, $H_3^{\perp}=\left\{\begin{array}{l}x+y+z=0\\x+y=0\end{array}\right\}$

$H_1\cap H_3 = \left\{2y+z=0,\ \underbrace{\begin{vmatrix}x&y&z\\1&1&1\\1&1&0\end{vmatrix}}_{-x+y}=0\right\}$, $(H_1\cap H_3)^{\perp}=\langle(0,2,1),(-1,1,0)\rangle$

$(H_1\cap H_3)^{\perp}+H_2 = \langle(0,2,1),(-1,1,0),(1,1,-2)\rangle$

10b) $B_0(H_1)=\left\{\left(0,\frac{1}{\sqrt5},\frac{-2}{\sqrt5}\right),(1,0,0)\right\}$, $B_0(H_2^{\perp})=\dots$, $B_0(H_3)=\dots$

10 c) $\overrightarrow{\text{Proy}^{\perp}_{H_1}(\vec{p})} = \left(0, \frac{2}{5}, -\frac{4}{5}\right)$ $\qquad$ $\text{Proy}^{\perp}_{H_3}(\vec{p}) = \vec{p}$

$\overrightarrow{\text{Proy}^{H_2}_{H_1}(\vec{p})}$ = "infinita" $\qquad$ $\overrightarrow{\text{Proy}^{H_2}_{H_3}(\vec{p})} = -\frac{1}{\sqrt{3}}\frac{(1,1,1)}{\sqrt{3}} + \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right)$

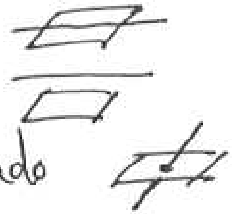
$\overrightarrow{\text{Proy}^{(0,0,1)}_{H_1}(\vec{p})} = (0,0,0)$ $\qquad$ $\text{Proy}^{(0,0,1)}_{H_3}(\vec{p})$ = "infinita"

10.d) $\widehat{(\vec{p}, H_1)} = \arccos\left(\frac{\vec{p}\cdot \text{Proy}^{\perp}_{H_1}}{\|\vec{p}\|\cdot\|\text{Proy}^{\perp}_{H_1}\|}\right) = \arccos\left(\frac{2}{\sqrt{5}}\right) \simeq 0'46$ rad

$\overrightarrow{d^{\perp}(\vec{p}, H_1)} = \left(0, -\frac{2}{5}, \frac{-1}{5}\right)$, $\|\overrightarrow{d^{\perp}(\vec{p}, H_1)}\| = \frac{1}{\sqrt{5}}$

10.e) $\cos(\widehat{H_1, H_3}) = \frac{(0,2,1)}{\sqrt{5}} \cdot \frac{(1,-1,0)}{\sqrt{2}} = \frac{-2}{\sqrt{10}} \Rightarrow (H_1, H_3) = \arccos\left(\frac{-2}{\sqrt{10}}\right)$

10.f) $\left.\begin{array}{r} 2y+z=0 \\ x-y=0 \\ by-z=-a \end{array}\right\}$
- $b=-2,\ a=0$ Comp. indet
- $b=-2,\ a\neq 0$ incomp.
- $b\neq -2$ comp. determinado



11) $\begin{vmatrix} 3 & 2 & 5 \\ 1 & 2 & 7 \\ 0 & 0 & 1 \end{vmatrix} = 4$

12) $5x+2y-z=-22$

13) $3x+2y+6z=6$

14) $R_1 \left\{\begin{array}{l} x-2y+z=2 \\ x-y=0 \end{array}\right.$ $\quad$ $\left.\begin{array}{l} P_0=(1,1,3) \\ P_0'=(2,2,4) \end{array}\right\}$ $\vec{v} = P_0' - P_0 = (1,1,1)$ vector director

$\hat{v} = (1,1,1)/\sqrt{3}$

$P=(1,1,1)$ $\qquad$ $P_0, P_0' \in R_1$

$\overrightarrow{d^{\perp}(P,R_1)} = \overrightarrow{P_0P} - (\overrightarrow{P_0P}\cdot\hat{v})\hat{v} = (0,0,-2) - \left\{(0,0,-2)\cdot\frac{(1,1,1)}{\sqrt{3}}\right\}\frac{(1,1,1)}{\sqrt{3}} = \frac{(2,2,-4)}{3}$

$\|d^{\perp}(P,R_1)\| = \frac{1}{3}\sqrt{4+4+16} = \frac{4}{3}\sqrt{2}$

$R_2 \left\{\begin{array}{l} x=5+3\lambda \\ y=-2+5\lambda \\ z=\lambda \end{array}\right\}$ $\quad$ $P_0=(5,-2,0)$

$\vec{v}=(3,5,1) \Rightarrow \hat{v} = \frac{(3,5,1)}{\sqrt{9+25+1}}$

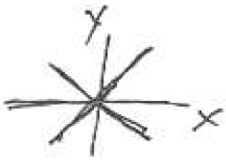
$\overrightarrow{d^{\perp}(P,R_2)} = \overrightarrow{P_0P} - (\overrightarrow{P_0P}\cdot\hat{v})\hat{v}$

$= (-4,3,1) - \left\{(-4,3,1)\frac{(3,5,1)}{\sqrt{35}}\right\}\cdot\frac{(3,5,1)}{\sqrt{35}}$

15) $\frac{17}{7}$ $\qquad$ 16) $\arccos\left(\frac{4}{21}\right) = 1'38$ rad.

17.a) $f(x,y)=\frac{x+y}{\sqrt{x^2+y^2}}$ $\quad x=r\cos\theta,\ y=r\,\text{sen}\,\theta$

(a) $f(x,y)=\cos\theta+\text{sen}\,\theta=K \Rightarrow \theta=cte$



"escalera caracol"

(bi) $D_{\hat{h}}f(-1,2)=0 \quad D_{\hat{h}}f(1,-3)=\frac{\sqrt{2}}{25}$

(bii) $\overrightarrow{agua}(-1,2)=-\left(\frac{6}{5\sqrt{5}},\frac{3}{5\sqrt{5}}\right)$, $\overrightarrow{agua}(1,-3)=\frac{\sqrt{2}}{5\sqrt{5}}(3,1)$

(biii) $D_{max}f(-1,2)=\frac{3}{5} > D_{max}f(1,-3)=\frac{2}{5} \Rightarrow$ más rápido en $(-1,2)$

(c) $z-\frac{1}{\sqrt{5}}=\frac{6}{5\sqrt{5}}(x+1)+\frac{3}{5\sqrt{5}}(y-2)$

17.b) (a) $f(x,y)=\frac{x^2}{\sqrt{1-2x^2-y}}=K \Rightarrow x^4=K^2(1-2x^2-y) \Rightarrow \boxed{y=1-2x^2-\frac{x^4}{K^2}}$

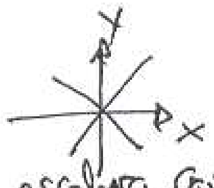
(bi) $D_{\hat{h}}f(-1,2)=-\frac{1}{\sqrt{5}} \quad D_{\hat{h}}f(1,-3)=\frac{\sqrt{3/10}}{2}$

(bii) $\overrightarrow{agua}(-1,2)=-\left(0,\frac{1}{2}\right)$, $\overrightarrow{agua}(1,-3)=-\left(\frac{5}{3\sqrt{6}},\frac{1}{12\sqrt{6}}\right)$

(biii) $D_{max}f(-1,2)=\frac{1}{2} < D_{max}f(1,-3)=\frac{\sqrt{401/6}}{12}$

(c) $z-1=\cancel{0\cdot(x+1)}+\frac{1}{2}(y-2)$

17.c) (a) $f(x,y)=\text{arctg}(y/x)=\text{arctg}(\text{tg}\,\theta)=\theta=cte$



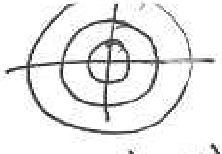
escalera caracol.

(bi) $D_{\hat{h}}f(-1,2)=0$, $D_{\hat{h}}f(1,-3)=\frac{1}{10\sqrt{5}}$

(bii) $\overrightarrow{agua}(-1,2)=-\left(-\frac{2}{5},-\frac{1}{5}\right)$, $\overrightarrow{agua}(1,-3)=-\left(\frac{3}{10},\frac{1}{10}\right)$

(biii) $D_{max}f(-1,2)=\frac{1}{\sqrt{5}} > D_{max}f(1,-3)=\frac{1}{\sqrt{10}}$

(c) $z+\text{arctg}(2)=-\frac{2}{5}(x+1)-\frac{1}{5}(y-2)$

17.d) (a) $f(x,y) = \frac{e^{-r}}{r} = cte \Rightarrow r = cte \Rightarrow$  circunferencias

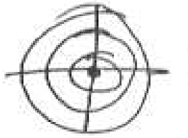
(bi) $D_{\vec{h}} f(-1,2) = \frac{1}{5}(1+\sqrt{5})e^{-\sqrt{5}}$, $D_{\vec{h}} f(1,-3) = \frac{-7}{100}(\sqrt{2}+2\sqrt{5})e^{-\sqrt{10}}$

(bii) $\overrightarrow{agua}(-1,2) = \left(\frac{1}{25}(5+\sqrt{5})e^{-\sqrt{5}}, -\frac{2}{25}(5+\sqrt{5})e^{-\sqrt{5}}\right)$

$\overrightarrow{agua}(1,-3) = -\left(-\frac{1}{100}(10+\sqrt{10})e^{-\sqrt{10}}, \frac{3}{100}(10+\sqrt{10})e^{-\sqrt{10}}\right)$

(biii) $D_{max} f(-1,2) = \frac{1}{5}\sqrt{2(3+\sqrt{5})}\, e^{-\sqrt{5}} > D_{max} f(1,-3) = \frac{1}{10}\sqrt{11+2\sqrt{10}}\, e^{-\sqrt{10}}$

(c) $z - \frac{e^{-\sqrt{5}}}{\sqrt{5}} = \frac{1}{25}(5+\sqrt{5})e^{-\sqrt{5}}(x+1) - \frac{2}{25}(5+\sqrt{5})e^{-\sqrt{5}}(y-2)$

17.e) (a) $f(x,y) = \frac{\operatorname{sen} r}{r^2} = cte \Rightarrow r = cte \Rightarrow$  Circunf.

(bi) $D_{\vec{h}} f(-1,2) = \frac{1}{25}\left(-5\cos(\sqrt{5}) + 2\sqrt{5}\operatorname{sen}(\sqrt{5})\right)$

$D_{\vec{h}} f(1,-3) = \frac{7}{500}\left(5\sqrt{2}\cos\sqrt{10} - 2\sqrt{5}\operatorname{sen}\sqrt{10}\right)$

(bii) $\overrightarrow{agua}(-1,2) = -\left[\frac{1}{25}(-\sqrt{5}\cos\sqrt{5} + 2\operatorname{sen}\sqrt{5}), \frac{2}{25}(\sqrt{5}\cos\sqrt{5} - 2\operatorname{sen}\sqrt{5})\right]$

$\overrightarrow{agua}(1,-3) = -\left[\frac{1}{100}(\sqrt{10}\cos\sqrt{10} - 2\operatorname{sen}\sqrt{10}), \frac{1}{100}(-3\sqrt{10}\cos\sqrt{10} + 6\operatorname{sen}\sqrt{10})\right]$

(bii) $D_{max} f(-1,2) = \frac{1}{5}\sqrt{\frac{1}{10}\left(9+\cos(2\sqrt{5}) - 4\sqrt{5}\operatorname{sen}(2\sqrt{5})\right)}$

$D_{max} f(1,-3) = \frac{1}{10}\sqrt{\frac{1}{10}\left(7+3\cos(2\sqrt{10}) - 2\sqrt{10}\operatorname{sen}(2\sqrt{10})\right)}$

(c) $z - \frac{\operatorname{sen}\sqrt{5}}{\sqrt{5}} = \frac{1}{25}(-\sqrt{5}\cos\sqrt{5} + 2\operatorname{sen}\sqrt{5})(x+1) + \frac{2}{25}(\sqrt{5}\cos\sqrt{5} - 2\operatorname{sen}\sqrt{5})(y-2)$

17.f) (a) $f(x,y)=\sqrt{1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2}=K \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1-K^2$ elipses para $|K|<1$

Tomemos $a=3$, $b=4$, por ejemplo:

(bi) $D_{\hat{h}} f(1,2)=\frac{13}{6\sqrt{115}}$, $D_{\hat{h}} f(1,-3)=\frac{-7\sqrt{5/47}}{6}$

(bii) $\overrightarrow{agua}(-1,2) = -\left(\frac{2}{3\sqrt{23}}, \frac{-3}{4\sqrt{23}}\right)$

$agua(1,-3) = -\left(\frac{-4}{3\sqrt{47}}, \frac{9}{4\sqrt{47}}\right)$

(biii) $D_{max} f(-1,2) = \frac{\sqrt{\frac{145}{23}}}{12} < D_{max} f(1,-3) = \frac{\sqrt{\frac{985}{47}}}{12}$

(c) $z - \frac{\sqrt{23}}{6} = \frac{2}{3\sqrt{23}}(x+1) - \frac{3}{4\sqrt{23}}(y-2)$

18) $\left.\begin{array}{l} x = 10 \pm \overbrace{0'1}^{dx} \\ y = 20 \pm \underbrace{0'1}_{dy} \end{array}\right\}$ $\boxed{A = 10 \cdot 20 \pm dA(10,20) = 200 \pm 3}$

$dA(10,20) = \underbrace{\left|\frac{\partial A(10,20)}{\partial x}\right|}_{y} dx + \underbrace{\left|\frac{\partial A(10,20)}{\partial y}\right|}_{x} dy = 20 \cdot 0'1 + 10 \cdot 0'1 = 3$

19) $\left.\begin{array}{l} V = V_0 \pm dV = 3 \pm 0'01 \\ T = T_0 \pm dT = 300 \pm 5 \\ N = 1 \end{array}\right\}$ $P = NR\frac{T}{V}$ $\quad \frac{\partial P}{\partial T} = NR\frac{1}{V}$, $\quad \frac{\partial P}{\partial V} = -NR\frac{T}{V^2}$

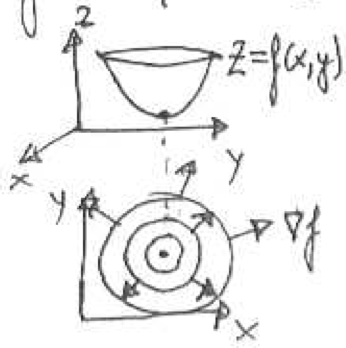
$dP(3,300) = \left|\frac{\partial P(3,300)}{\partial V}\right| dV + \left|\frac{\partial P(3,300)}{\partial T}\right| dT = \frac{R}{3^2}300 \cdot 0'01 + \frac{R}{3}5$

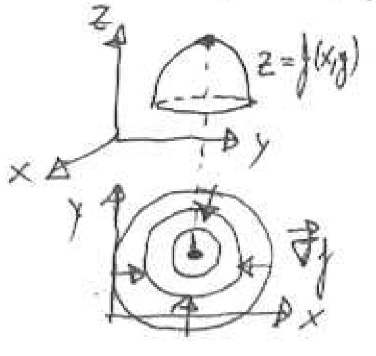
$= \frac{R}{3} + \frac{R}{3} \cdot 5 = 2R$

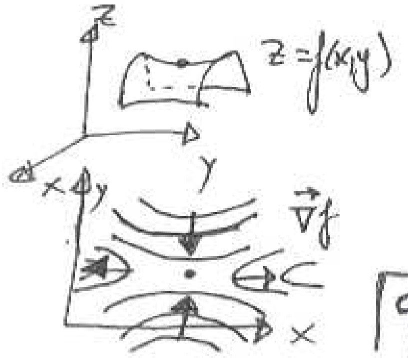
$\boxed{P = R\frac{300}{3} \pm 2R = 100R \pm 2R}$

20) $y = -\frac{1}{4}x + \frac{1}{2}$ 21) $z = -\frac{1}{10}x - \frac{1}{4}y + \frac{1}{2}$

22)

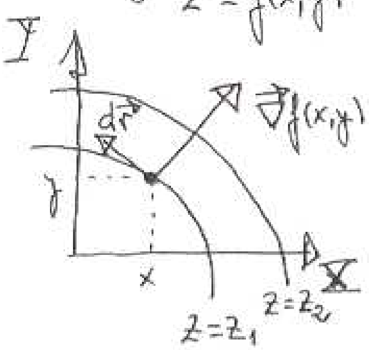






El gradiente es perpendicular a las curvas de nivel. Demostración:

$z = f(x,y)$



Diferencial: $\begin{cases} df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \vec{\nabla} f(x,y) \bullet d\vec{r} \\ \vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right), \; d\vec{r} = (dx, dy) \end{cases}$

si $d\vec{r}$ es tangente a la curva de nivel $z=z_1$ entonces $df(x,y) \overset{!}{=} 0 \Rightarrow \vec{\nabla} f(x,y) \cdot d\vec{r} = 0 \Rightarrow$

$\Rightarrow \vec{\nabla} f(x,y) = \vec{0}$ ó bien $\vec{\nabla} f(x,y) \perp d\vec{r} \Rightarrow \vec{\nabla} f(x,y)$ es perpendicular a la curva de nivel $z=z_1$ en el punto (x,y).

23) $M_{B_c}(T) = \begin{pmatrix} 2 & 1/3 \\ 1/3 & 3/2 \end{pmatrix}$ $\begin{cases} \lambda_+ = 4/3 & \vec{v}_+ = (-\frac{1}{2}, 1) \\ \lambda_- = 13/3 & \vec{v}_- = (2, 1) \end{cases}$

$|M| > 1$ dilatación

24) $M_{B_c}(T) = \begin{pmatrix} 1 & 1/2 \\ 0 & 5/4 \end{pmatrix}$ $\begin{cases} \lambda_+ = 1 & \vec{v}_+ = (1,0) \\ \lambda_- = 5/4 & \vec{v}_- = (2,1) \end{cases}$

$|M| > 1$ dilat.

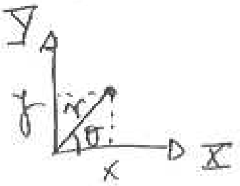
25) $M_{B_c}(T) = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ $\begin{cases} \lambda_+ = -2 & \vec{v}_+ = (-1,1) \\ \lambda_- = 4 & \vec{v}_- = (1,1) \end{cases}$

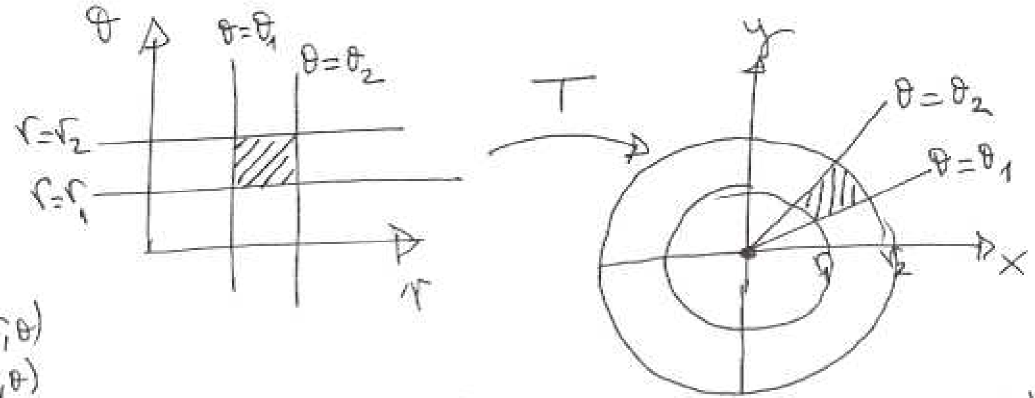
26) $M_{B_c}(T) = \begin{pmatrix} 1'1 & 7/30 \\ 0 & 20/15 \end{pmatrix}$ $\begin{cases} \lambda_+ = 1 & \vec{v}_+ = (1,1) \\ \lambda_- = 1'1 & \vec{v}_- = (1,0) \end{cases}$

$M_{B_c}(T) = \begin{pmatrix} 1'003 & -0'001 \\ -0'002 & 1'002 \end{pmatrix}$ $\begin{cases} \lambda_+ = 1'004 & \vec{v}_+ = (1,-1) \\ \lambda_- = 1'001 & \vec{v}_- = (1,2) \end{cases}$

28) $M_{B_c}(T) = \begin{pmatrix} 1'1 & 0'2 & 0'3 \\ 0'1 & 1'2 & 0'4 \\ 0'4 & 0'3 & 1'3 \end{pmatrix}$ dilatación

29)





$x = r\cos\theta = x(r,\theta)$

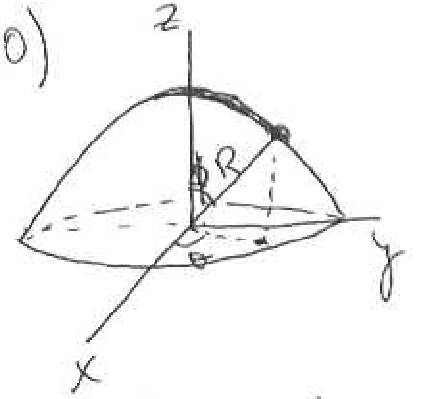
$y = r \operatorname{sen}\theta = y(r,\theta)$

$$\begin{pmatrix} dx(r,\theta) \\ dy(r,\theta) \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial r}dr + \frac{\partial x}{\partial \theta}d\theta \\ \frac{\partial y}{\partial r}dr + \frac{\partial y}{\partial \theta}d\theta \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix}}_{\begin{pmatrix} \cos\theta & -r\operatorname{sen}\theta \\ \operatorname{sen}\theta & r\cos\theta \end{pmatrix}} \begin{pmatrix} dr \\ d\theta \end{pmatrix}$$

Matriz Jacobiana

$$\begin{vmatrix} \cos\theta & -r\operatorname{sen}\theta \\ \operatorname{sen}\theta & r\cos\theta \end{vmatrix} = \boxed{r}$$ Jacobiano

30)



$x(R,\phi,\theta) = R\operatorname{sen}\phi\cos\theta$

$y(R,\phi,\theta) = R\operatorname{sen}\phi\operatorname{sen}\theta$

$z(R,\phi,\theta) = R\cos\phi$

Matriz Jacobiana $JT(R,\phi,\theta) = \begin{pmatrix} \frac{\partial x}{\partial R} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial R} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial \theta} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial \theta} \end{pmatrix} =$

$$= \begin{pmatrix} \operatorname{sen}\phi\cos\theta & R\cos\phi\cos\theta & -R\operatorname{sen}\phi\operatorname{sen}\theta \\ \operatorname{sen}\phi\operatorname{sen}\theta & R\cos\phi\operatorname{sen}\theta & R\operatorname{sen}\phi\cos\theta \\ \cos\phi & -R\operatorname{sen}\phi & 0 \end{pmatrix}$$

Jacobiano $|JT(R,\phi,\theta)| = R^2\operatorname{sen}\phi$

31) $\vec{\nabla} f(x,y) = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath}$, $df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = df(r,\theta) = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial \theta}d\theta$

$$df(x,y) = \frac{\partial f}{\partial x}\left(\frac{\partial x}{\partial r}dr + \frac{\partial x}{\partial \theta}d\theta\right) + \frac{\partial f}{\partial y}\left(\frac{\partial y}{\partial r}dr + \frac{\partial y}{\partial \theta}d\theta\right) = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial \theta}d\theta \Rightarrow$$

$$\left.\begin{aligned} \frac{\partial f}{\partial r} &= \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} \\ \frac{\partial f}{\partial \theta} &= \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \frac{\partial f}{\partial x} &= \cos\theta\frac{\partial f}{\partial r} - \frac{1}{r}\operatorname{sen}\theta\frac{\partial f}{\partial \theta} \\ \frac{\partial f}{\partial y} &= \operatorname{sen}\theta\frac{\partial f}{\partial r} + \frac{1}{r}\cos\theta\frac{\partial f}{\partial \theta} \end{aligned}\right\} \Rightarrow$$

$$\boxed{\vec{\nabla} f(x,y) = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} = \cdots = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta}}$$

$\hat{r} = \cos\theta\,\hat{\imath} + \operatorname{sen}\theta\,\hat{\jmath}$

$\hat{\theta} = -\operatorname{sen}\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$