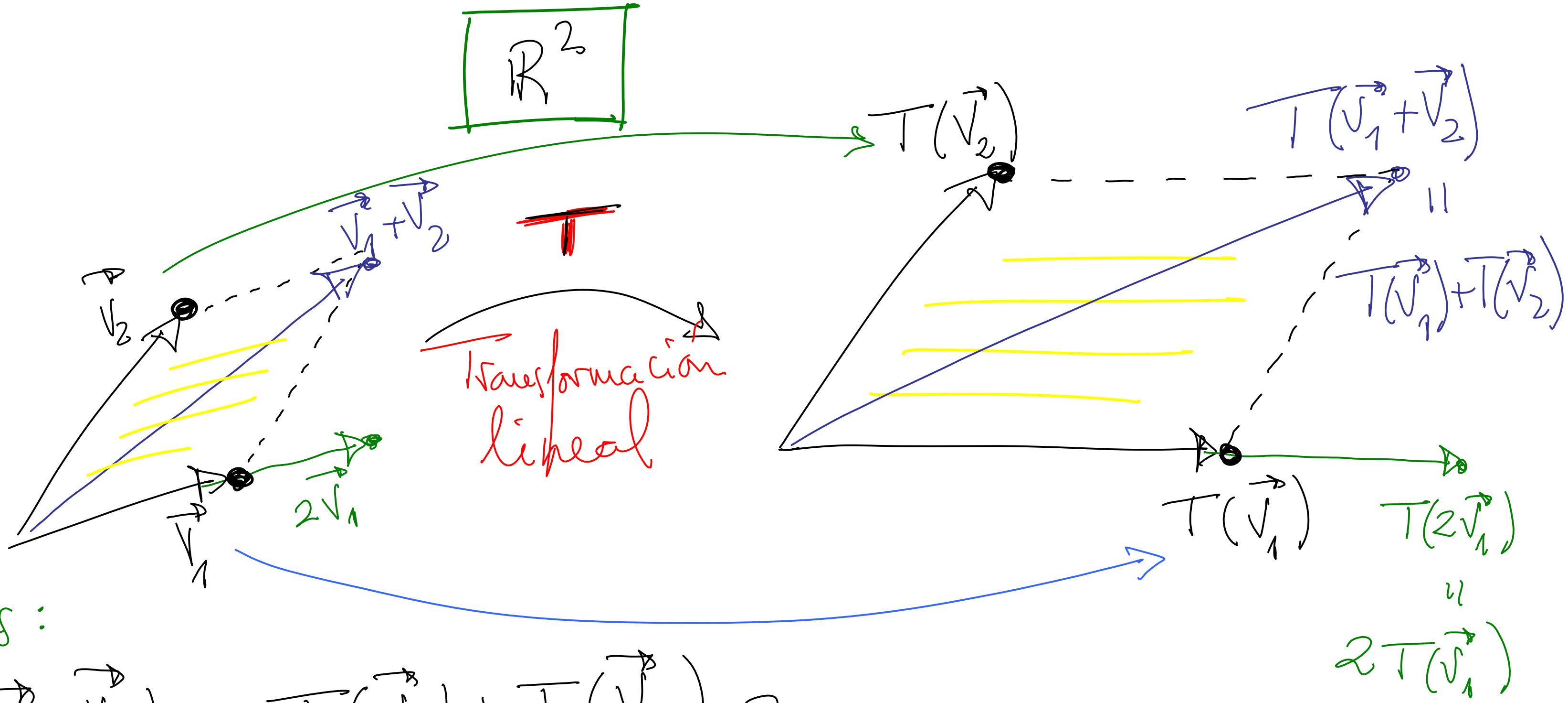
TRANSFORMACIONES LINEALES Y DIAGONALIZACIÓN

Título de la nota 21/11/2008

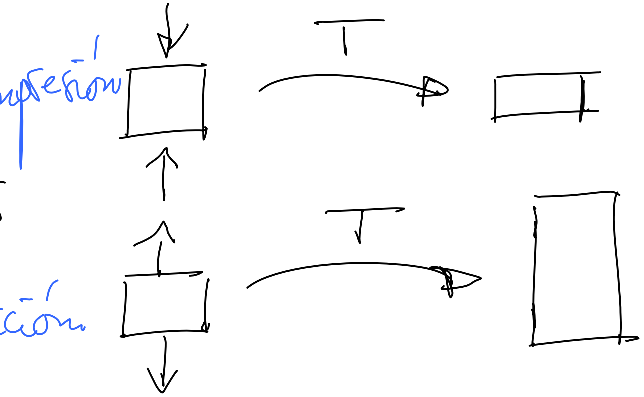


Propiedades:

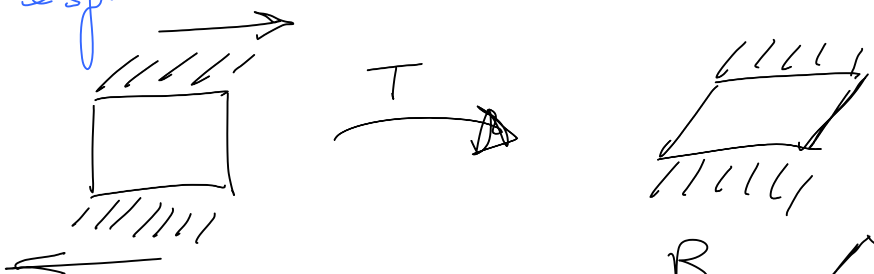
$$\left.\begin{array}{l} \text{P.1)}\quad T(\vec{v}_1+\vec{v}_2) = T(\vec{v}_1)+T(\vec{v}_2) \\ \text{P.2)}\quad T(\alpha\vec{v}_1) = \alpha\, T(\vec{v}_1) \end{array}\right\} \underline{T \text{ es lineal}}$$

Casos sencillos:

1) DILATACIONES Y CONTRACCIONES

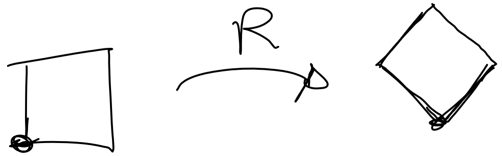


esfuerzo "cortante"

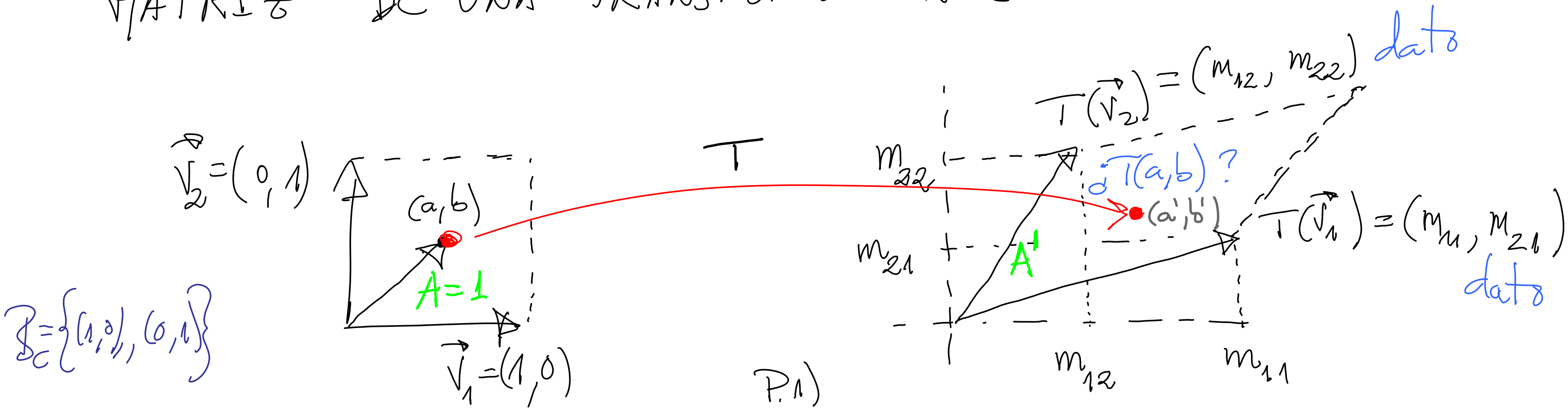
2) 

T: CIZALLADURA

3) ROTACIÓN



MATRIZ DE UNA TRANSFORMACIÓN LINEAL EN LA BASE CANÓNICA

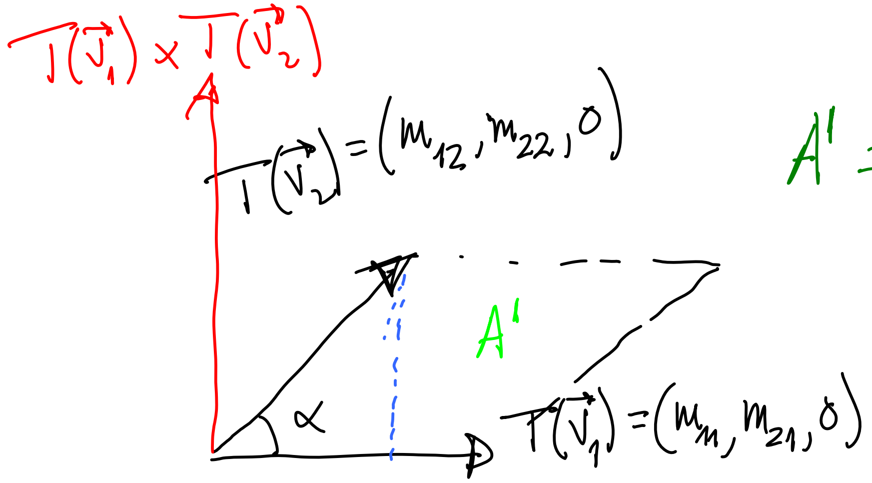


$$T(a,b) = T\big(a(1,0)+b(0,1)\big) \overset{P.1)}{=} T\big(a(1,0)\big) + T\big(b(0,1)\big) =$$

$$= a\underbrace{T(\overbrace{1,0}^{\vec{v}_1})}_{(m_{11},\,m_{21})} + b\underbrace{T(\overbrace{0,1}^{\vec{v}_2})}_{(m_{12},\,m_{22})} = \big(\underbrace{a\,m_{11}+b\,m_{12}}_{a'},\ \underbrace{a\,m_{21}+b\,m_{22}}_{b'}\big)$$

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \underbrace{\begin{array}{cc} T(1,0) & T(0,1) \\ \multicolumn{2}{c}{\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}_{2\times 2}} \end{array}}_{M_{B_c}(T)} \begin{pmatrix} a \\ b \end{pmatrix}_{2\times 1} = \begin{pmatrix} m_{11}a + m_{12}\cdot b \\ m_{21}a + m_{22}\,b \end{pmatrix}_{2\times 1}$$

$\longleftarrow$ matriz de la transformación lineal T en la base canónica B_c



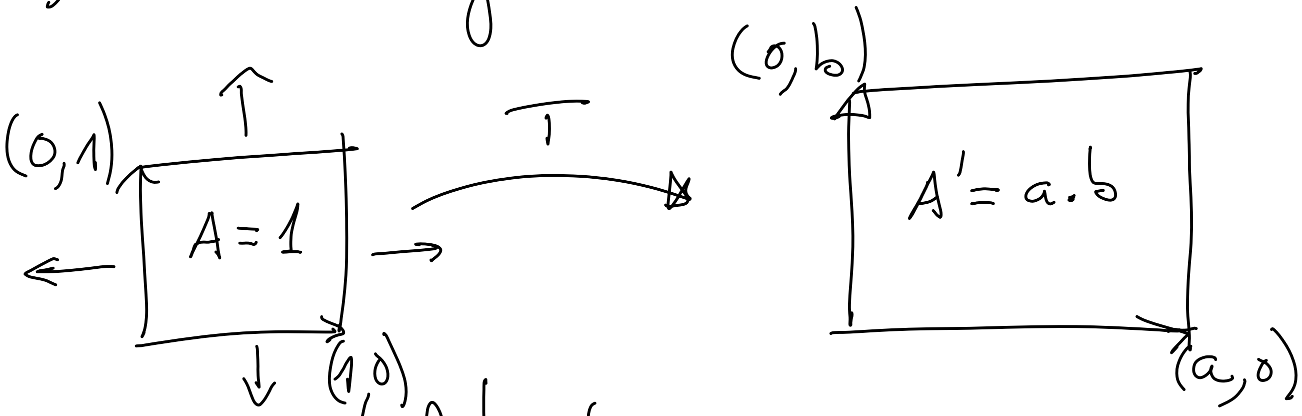
$$A' = \|T(\vec{v}_1) \times T(\vec{v}_2)\| = \underbrace{\|T(\vec{v}_1)\|}_{b} \cdot \underbrace{\|T(\vec{v}_2)\| \cdot \operatorname{sen} \alpha}_{h}$$

$$T(\vec{v}_1) \times T(\vec{v}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ m_{11} & m_{21} & 0 \\ m_{12} & m_{22} & 0 \end{vmatrix} = \hat{k} \underbrace{\begin{vmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{vmatrix}}_{|M_{B_c}(T)|} \rightarrow$$

$$A' = \|T(\vec{v}_1) \times T(\vec{v}_2)\| = |M_{B_c}(T)|$$

Matrices de transformaciones sencillas:

1) Dilatación y contracción



$$M_{B_c}(T) = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = M$$

$$A' = |M| = a \cdot b$$

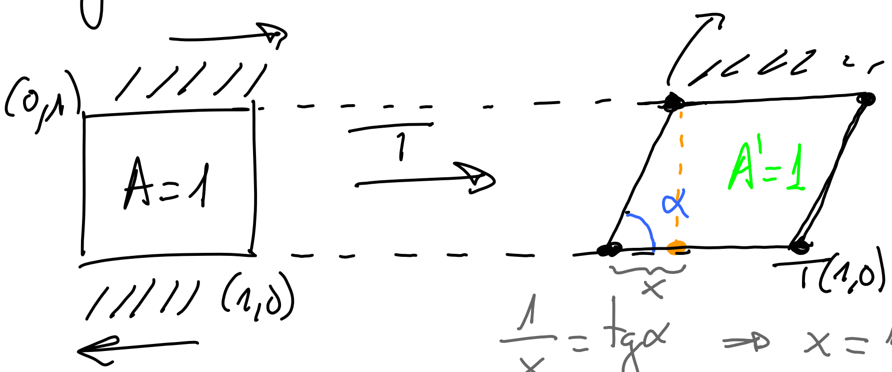
A' $> 1 \longrightarrow$ dilatación

A' $< 1 \longrightarrow$ contracción

A' $= 1 \longrightarrow$ se queda igual

2) Cizalladura

$T(0,1) = (\text{cotg}(\alpha), 1)$

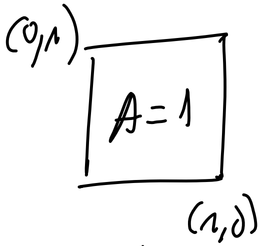


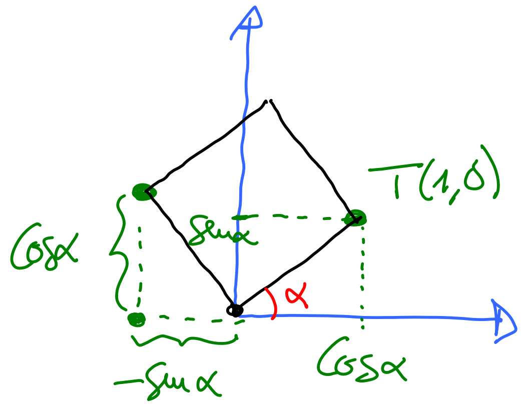
$T(1,0) = (1,0)$

$$\frac{1}{x} = \text{tg}\,\alpha \Rightarrow x = 1/\text{tg}(\alpha) = \text{cotg}(\alpha)$$

$$M_{B_c}(T) = \begin{array}{c} \begin{pmatrix} 1 & \operatorname{Cotg}(\alpha) \\ 0 & 1 \end{pmatrix} \\ \begin{array}{cc} T(1,0) & T(0,1) \end{array} \end{array} \qquad A' = |M| = 1$$

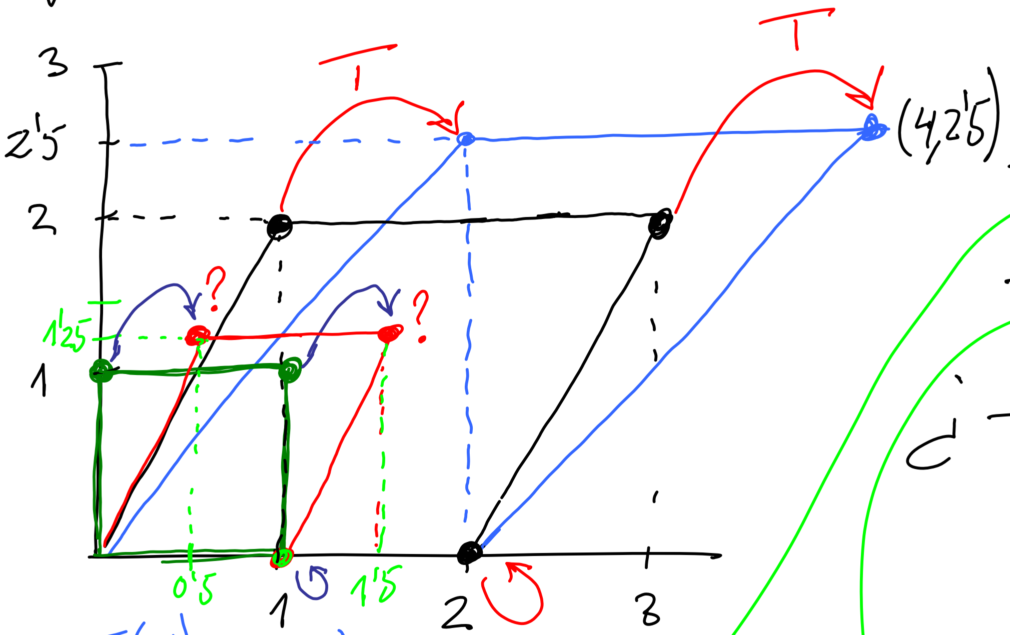
3) Rotación





$$M_{B_c}(T) = \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha \\ \operatorname{sen}\alpha & \cos\alpha \end{pmatrix} = M, \qquad A' = |M| = \cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

Ejercicio 24 de la relación de problemas



Datos :

$T(2,0) = (2,0)$ dato

$T(1,2) = (2, 2'5)$ dato

¿$T(0,1)$? , ¿$T(1,0)$?

$$\underbrace{\begin{array}{cc} T(1,0) & T(0,1) \\ \multicolumn{2}{c}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \end{array}}_{M(T)_{B_c}} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad , \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2'5 \end{pmatrix}$$

$$\left.\begin{array}{l} a + 2b = 2 \\ c + 2d = 2'5 \end{array}\right\} \quad \begin{array}{l} b = 1/2 \\ d = \dfrac{2'5}{2} = \dfrac{5}{4} \end{array}$$

$$\left.\begin{aligned} 2a &= 2 \\ 2c &= 0 \end{aligned}\right\} \begin{aligned} a &= 1 \\ c &= 0 \end{aligned}$$

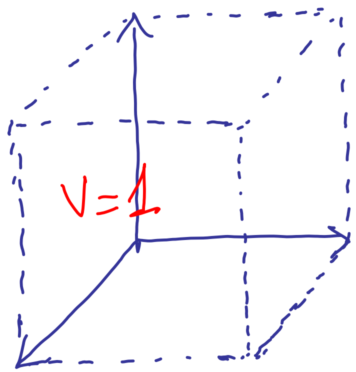
$$\mathcal{M}_{B_c}(T) = \begin{pmatrix} 1 & 1/2 \\ 0 & \frac{5}{4} \end{pmatrix}$$

$$A' = |M| = \frac{5}{4} > 1$$

dilatación

$$\boxed{\mathbb{R}^3}$$

$\vec{v}_3 = (0,0,1)$



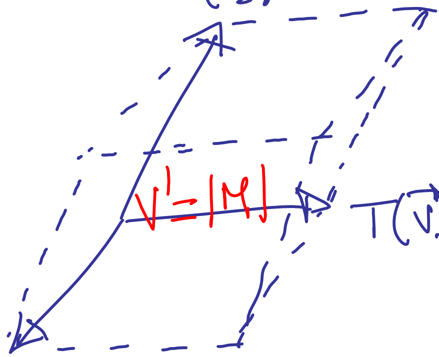
$V = 1$

$\vec{v}_2 = (0,1,0)$

$\vec{v}_1 = (1,0,0)$

$\xrightarrow{T}$

$T(\vec{v}_3) = (m_{13}, m_{23}, m_{33})$



$V' = |M|$

$T(\vec{v}_2) = (m_{12}, m_{22}, m_{32})$

$T(\vec{v}_1) = (m_{11}, m_{21}, m_{31})$

$$\mathcal{M}_{B_c}(T) = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc} T(1,0,0) & T(0,1,0) & T(0,0,1) \\ \end{array}$$

$$\mathcal{M}_{B_c}(T) = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = M$$

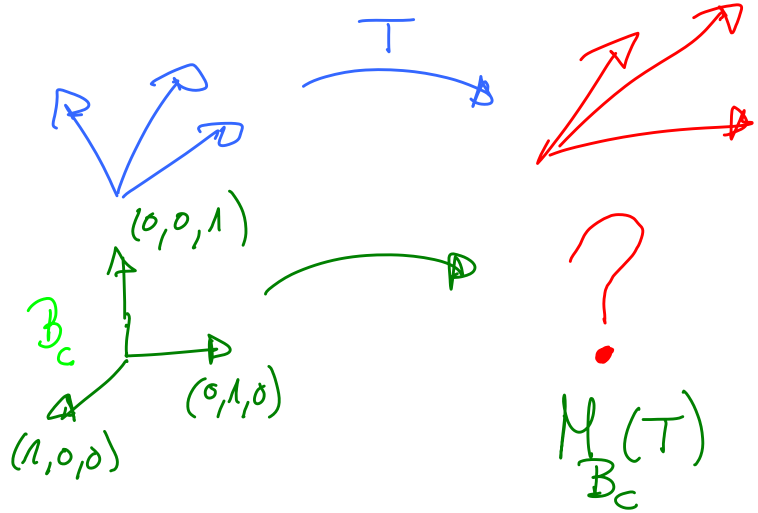
Examen de Junio 2008. Tipo A | Sea

$T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ dada por:

1) $T(1,-2,3) = \frac{1}{3}(-17,-24,-19)$

2) $T(0,2,3) = \frac{1}{3}(-4,-8,-7)$

3) $T(0,0,3) = (-4,-6,-5)$



1) $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -17 \\ -24 \\ -19 \end{pmatrix} \Longrightarrow$

$$\left.\begin{aligned} 3a - 6b + 9c &= -17 \\ 3d - 6e + 9f &= -24 \\ 3g - 6h + 9i &= -19 \end{aligned}\right\} \quad \begin{aligned} a &= 1 \\ d &= 4/3 \\ g &= 4/3 \end{aligned}$$

2) $\begin{pmatrix} & & \\ & " & \\ & & \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -4 \\ -8 \\ -7 \end{pmatrix} \Longrightarrow$

$$\left.\begin{aligned} 6b + 9c &= -4 \\ 6e + 9f &= -8 \\ 6h + 9i &= -7 \end{aligned}\right\} \quad \left.\begin{aligned} b &= 4/3 \\ e &= (-8+18)/6 = \frac{10}{6} = \frac{5}{3} \\ h &= (-7+15)/6 = 4/3 \end{aligned}\right\}$$

3) $\begin{pmatrix} & \| & \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -6 \\ -5 \end{pmatrix} \Rightarrow \left.\begin{array}{l} 3c = -4 \\ 3f = -6 \\ 3i = -5 \end{array}\right\} \left.\begin{array}{l} c = -4/3 \\ f = -2 \\ i = -5/3 \end{array}\right\}$

$$M_{B_c}(T) = \frac{1}{3}\begin{pmatrix} 3 & 4 & -4 \\ 4 & 5 & -6 \\ 4 & 4 & -5 \end{pmatrix}$$