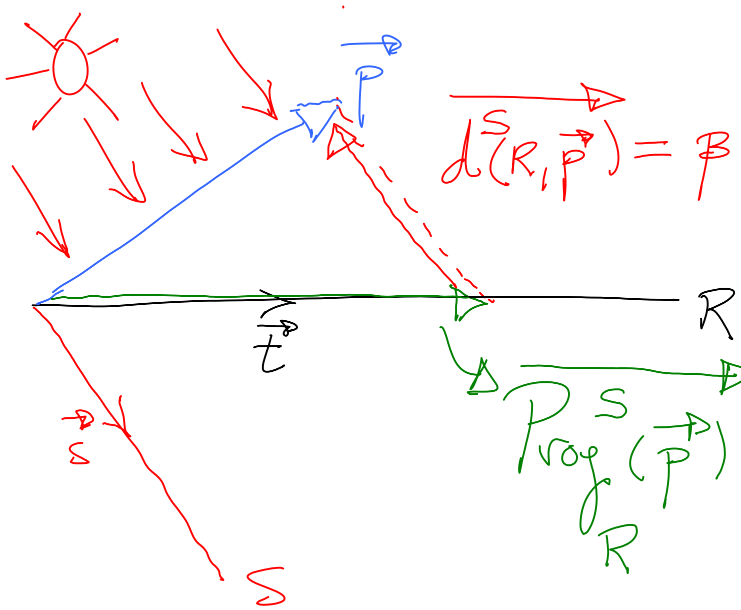
PROYECCIONES Y DISTANCIAS

Título de la nota 12/11/2008

A) Proyección de un vector $\vec{p}$ sobre una recta $R=\langle \vec{t}\rangle$ en la dirección de otra recta $S=\langle \vec{s}\rangle$

Datos: $\vec{p}, \vec{t}, \vec{s}$



$$\overrightarrow{d^S(R,\vec{p})} = \beta \vec{s}$$

$$\overrightarrow{\mathrm{Proy}^S_R(\vec{p})} = \alpha \vec{t} \, , \quad \boxed{\vec{p} = \underbrace{\alpha \vec{t}}_{\overrightarrow{\mathrm{Proy}^S_R(\vec{p})}} + \underbrace{\beta \vec{s}}_{\overrightarrow{d^S(\vec{p},R)}}}$$

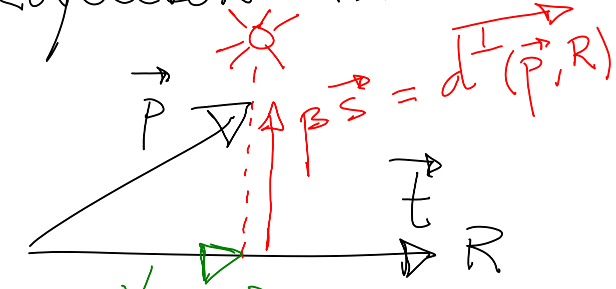
Ej: $\vec p=(1,2)$, $\vec t=(1,0)$, $\vec s=(1,-1)$

$$\vec p=\alpha\vec t+\beta\vec s \Rightarrow (1,2)=\alpha(1,0)+\beta(1,-1)\Rightarrow \begin{cases}1=\alpha+\beta\\ 2=-\beta\end{cases}$$

$$\beta=-2,\ \alpha=3 \Rightarrow \mathrm{Proy}_{\langle\vec t\rangle}^{\langle\vec s\rangle}(\vec p)=\alpha\vec t=3(1,0)=(3,0)$$

$$d^{\langle\vec s\rangle}(\vec p,\langle\vec t\rangle)=\beta\vec s=-2(1,-1)=(-2,2)$$

PROYECCIÓN ORTOGONAL $\quad S=R^{\perp}\Rightarrow \vec s\cdot\vec t=0$



$$\mathrm{Proy}_R^{\perp}(\vec p)=\alpha\vec t$$

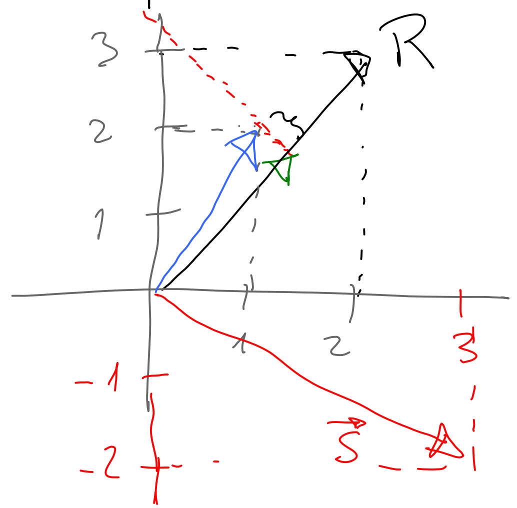
$$\vec p=\alpha\vec t+\beta\vec s$$

$$\vec p\cdot\vec t=\alpha\,\vec t\cdot\vec t+\cancel{\beta\,\vec s\cdot\vec t}^{\,0}=\alpha\,\vec t\cdot\vec t$$

$$\alpha=\frac{\vec p\cdot\vec t}{\vec t\cdot\vec t}$$

$$\boxed{\vec{\mathrm{Proy}}^{\perp}_{R}(\vec{p}) = \alpha\,\vec{t} = \frac{\vec{p}\cdot\vec{t}}{\underbrace{\vec{t}\cdot\vec{t}}_{\|\vec{t}\|^{2}}}\,\vec{t}} = \boxed{\vec{p}\cdot\hat{t}\;\hat{t}}$$

Ej] $\vec{p} = (1,2)$, $\vec{t} = (2,3)$ $\Rightarrow$ $\vec{\mathrm{Proy}}^{\perp}_{\langle\vec{t}\rangle}(\vec{p}) = (1,2)\cdot\dfrac{(2,3)}{\sqrt{13}}\,\dfrac{(2,3)}{\sqrt{13}} =$



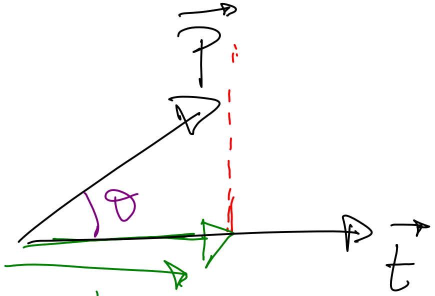
$\vec{s} = (3,-2)$

$$= \frac{2+6}{13}(2,3) = \frac{8}{13}(2,3)$$

$$\vec{d}^{\perp}(\vec{p},R) = \vec{p} - \vec{\mathrm{Proy}}^{\perp}_{R}(\vec{p}) =$$

$$= (1,2) - \frac{8}{13}(2,3) = \left(-\frac{3}{13}, \frac{2}{13}\right)$$

Distancia más corta de $\vec{p}$ a $\mathbb{R}$ $= \left\|\vec{d}^{\perp}(\vec{p},R)\right\| = \sqrt{\left(\frac{3}{13}\right)^2 + \left(\frac{2}{13}\right)^2}$



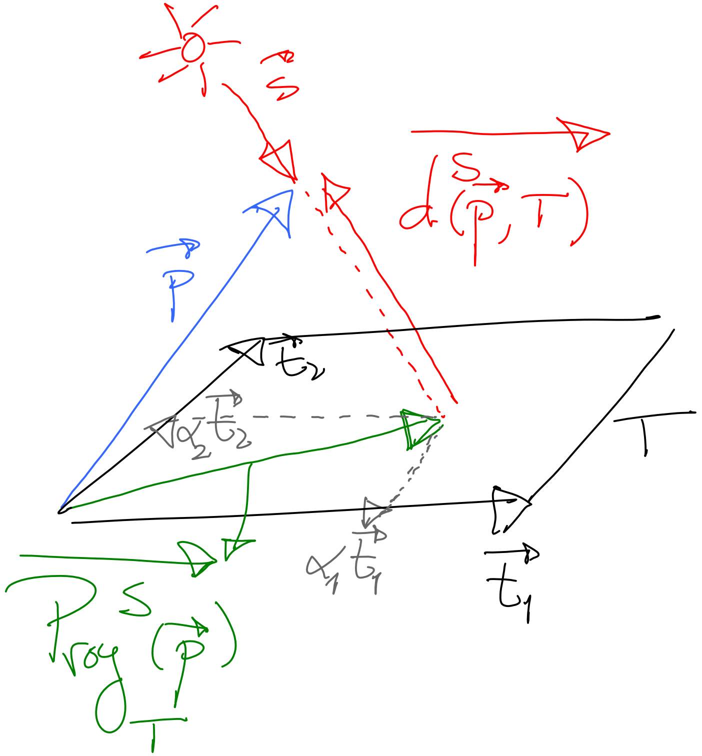
$$\mathrm{Proy}^{\perp}_{\langle \vec{t} \rangle}(\vec{p}) = \vec{p} \cdot \hat{t}\, \hat{t}$$

$$\cos\theta = \frac{\|\vec{p} \cdot \hat{t}\, \hat{t}\|}{\|\vec{p}\|} = \frac{|\vec{p} \cdot \hat{t}|}{\|\vec{p}\|} \cdot \frac{\|\vec{t}\|}{\|\vec{t}\|} = \frac{|\vec{p} \cdot \vec{t}\,|}{\|\vec{p}\| \cdot \|\vec{t}\|} \Rightarrow$$

$$\boxed{|\vec{p} \cdot \vec{t}\,| = \|\vec{p}\| \cdot \|\vec{t}\| \cdot \cos\theta}$$

$$\theta = \widehat{(\vec{t}, \vec{p})}$$

B] Proyección de un vector $\vec{p}$ sobre un plano $T = \langle \vec{t}_1, \vec{t}_2 \rangle$ en la dirección de una recta $S = \langle \vec{s} \rangle$



$$\vec{p} = \overrightarrow{Proy^S_T(\vec{p})} + \overrightarrow{d^S(\vec{p}, T)} =$$

$$= \alpha_1 \vec{t}_1 + \alpha_2 \vec{t}_2 + \beta \vec{s}$$

Ej] $\vec{p} = (1,1,1)$, $T = \{x+y+z=0\}$, $\vec{s} = \langle (1,1,0) \rangle$

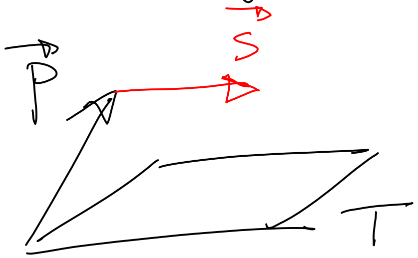
$\vec{t}_1 = (1,-1,0)$, $\vec{t}_2 = (0,1,-1)$

$$\vec p = \alpha_1 \vec t_1 + \alpha_2 \vec t_2 + \beta \vec s \Rightarrow (1,1,1) = \alpha_1 (1,-1,0) + \alpha_2 (0,1,-1) + \beta (1,1,0)$$

$$\underset{\vec p}{\begin{pmatrix}1\\1\\1\end{pmatrix}} = \left(\begin{array}{ccc} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 0 \\ \vec t_1 & \vec t_2 & \vec s \end{array}\right) \begin{pmatrix}\alpha_1\\ \alpha_2\\ \beta\end{pmatrix} \Rightarrow \begin{pmatrix}\alpha_1\\ \alpha_2\\ \beta\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 1\\ -1 & 1 & 1\\ 0 & -1 & 0\end{pmatrix}}_{A}^{-1} \begin{pmatrix}1\\1\\1\end{pmatrix}$$

a) $\text{rango}(A) = 3 \Leftrightarrow |A| \neq 0 \Leftrightarrow S \not\subset T$

b) $\text{rango}(A) = 2 \Rightarrow |A| = 0 \Rightarrow S \subset T$



Sistema incompatible (si $\vec p \notin T$)

"Sombra o proyección infinita"

Aplicando el Teorema de Rouché-Frobenius tenemos:

$A = (\vec{t}_1\ \vec{t}_2\ \vec{s})$ $\qquad \vec{p} = A \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \beta \end{pmatrix}$ $\qquad A' = (A \mid \vec{p})$ <u>matriz ampliada</u>

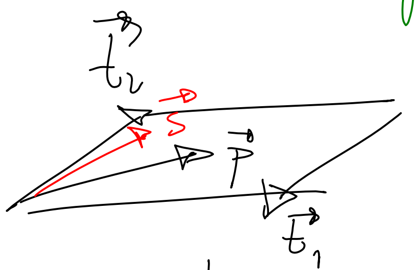
1) Sistema Compatible $\Leftrightarrow$ $\text{rango}(A) = \text{rango}(A')$

1.a) Determinado $\Leftrightarrow$ $\text{rango}(A) = 3 \Leftrightarrow \vec{s} \notin T$

Solución única

1.b) Indeterminado $\Leftrightarrow$ $\text{rango}(A) = 2 \Leftrightarrow \begin{cases} \vec{s} \in T \\ \vec{p} \in T \end{cases}$

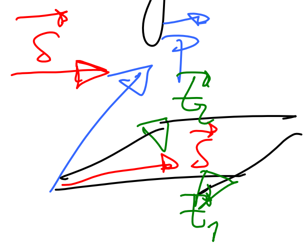
Muchas (infinitas soluciones)



S.C.I = "infinitas soluciones"

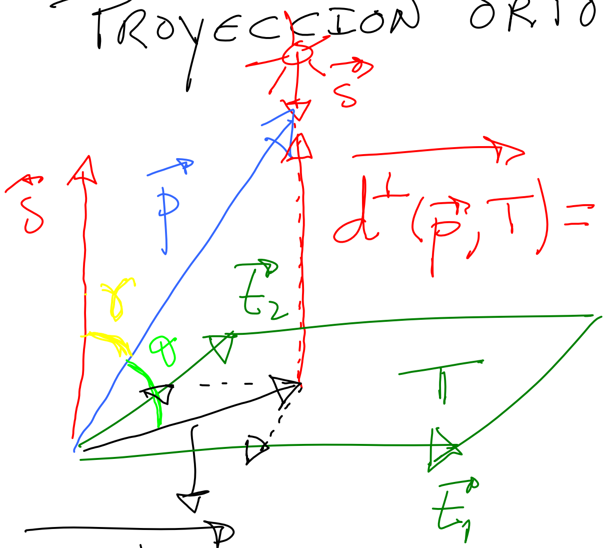
2) Sistema incompatible $\Leftrightarrow$ $\text{rango}(A) \neq \text{rango}(A')$

$\underbrace{\text{rango}(A) = 2}_{\vec{s} \in T} \neq \underbrace{\text{rango}(A') = 3}_{\vec{p} \notin T} \Rightarrow$



S.I.

PROYECCIÓN ORTOGONAL ($S \perp T$)



$$\overrightarrow{d^{\perp}(\vec p, T)} = \beta \vec s$$

$$\vec p = \overrightarrow{\text{Proy}^{\perp}_{T}(\vec p)} + \overrightarrow{d^{\perp}(\vec p, T)} = \alpha_1 \vec t_1 + \alpha_2 \vec t_2 + \beta \vec s$$

$$\vec p \cdot \vec s = \alpha_1 \underbrace{\vec t_1 \cdot \vec s}_{0} + \alpha_2 \underbrace{\vec t_2 \cdot \vec s}_{0} + \beta\, \vec s \cdot \vec s \Rightarrow$$

$$\beta = \frac{\vec p \cdot \vec s}{\vec s \cdot \vec s}$$

$$\overrightarrow{\text{Proy}^{\perp}_{T}(\vec p)} = \alpha_1 \vec t_1 + \alpha_2 \vec t_2$$

$$\boxed{\overrightarrow{\text{Proy}^{\perp}_{T}(\vec p)}} = \vec p - \overrightarrow{d^{\perp}(\vec p, T)} = \vec p - \beta \vec s = \boxed{\vec p - \frac{\vec p \cdot \vec s}{\vec s \cdot \vec s}\, \vec s}$$

Ej) Calcula la proyección ortogonal de $\vec p = (1,1,1)$ sobre

$T = \{(x,y,z) \in \mathbb{R}^3 : x + 2y - z = 0\}$. $\vec s = (1,2,-1)$

$$\overrightarrow{\mathrm{Proy}^{\perp}_{T}(\vec{p})} = \vec{p} - \beta\vec{s} = (1,1,1) - \frac{(1,1,1)\cdot(1,2,-1)}{(1,2,-1)\cdot(1,2,-1)}(1,2,-1) =$$

$$= (1,1,1) - \frac{2}{6}(1,2,-1) = \left(1-\frac{1}{3}, 1-\frac{2}{3}, 1+\frac{1}{3}\right) = \left(\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$$

Calcula la distancia de $\vec{p}$ a T

$$\left\| \overrightarrow{d^{\perp}(\vec{p}, T)} \right\| = \left\| \beta \vec{s} \right\| = \left\| \frac{2}{6}(1,2,-1) \right\| = \left\| \left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}\right) \right\| = \frac{1}{3}\sqrt{6}$$

Calcula el ángulo θ que forma $\vec{p}$ con T

$$\cos\theta = \frac{\left\| \overrightarrow{\mathrm{Proy}^{\perp}_{\parallel}(\vec{p})} \right\|}{\|\vec{p}\|} = \frac{\left\| \left(\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right) \right\|}{\|(1,1,1)\|} = \frac{\frac{1}{3}\sqrt{4+1+16}}{\sqrt{3}} = \frac{1}{3}\frac{\sqrt{21}}{\sqrt{3}} = \frac{1}{3}\sqrt{7}$$

otra forma

$$\vec{p}\cdot\vec{s} = \|\vec{p}\| \cdot \|\vec{s}\| \cdot \cos\gamma \Rightarrow \cos\gamma = \frac{\vec{p}\cdot\vec{s}}{\|\vec{p}\| \cdot \|\vec{s}\|} = \frac{2}{\sqrt{3}\sqrt{4}} = \frac{1}{\sqrt{3}}$$

$$\gamma = 90^{\circ} - \theta \Rightarrow \theta = 90^{\circ} - \gamma$$

$\boxed{\text{Examen de Septiembre 2008. Tipo A}}$

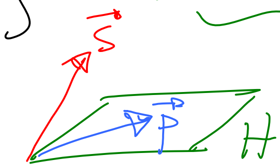
$H_1 = \langle (1,1,-1), (0,2,-1) \rangle$, $H_3 = \langle (1,3,-2) \rangle$

$H_2 = \{ x-2y-z=0 \}$, $H_4 = \left\{ \begin{array}{l} y-z=0 \\ 2x-3y-3z=0 \end{array} \right\}$

Calcula la $\text{Proy}_H^S(\vec{p})$ en los siguientes casos:

1) $\vec{p} = (-1,1,1)$, $H = H_3^{\perp}$, $S = H_2^{\perp} = \langle \underbrace{(1,-2,-1)}_{\vec{s}} \rangle$

$H = H_3^{\perp} = \{ 1 \cdot x + 3y - 2z = 0 \} = \langle \underbrace{(\qquad)}_{\vec{t}_1} , \underbrace{(\qquad)}_{\vec{t}_2} \rangle$



$\vec{p} \overset{?}{\in} H$: $-1+3-2=0$ sí

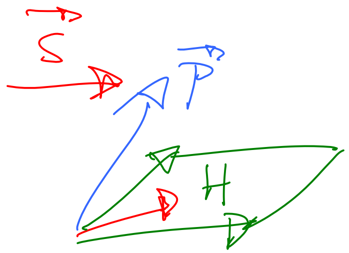
$\vec{s} \overset{?}{\in} H$: $1-6+2 \neq 0$ no $\longrightarrow$ $\text{Proy}_H^S(\vec{p}) = \vec{p} = \boxed{(-1,1,1)}$

2) $\vec{p} = (1,1,1)$, $H = H_1$, $S = H_3$

$H = H_1 = \langle (1,1,-1), (0,2,-1) \rangle$, $S = \langle (1,3,-2) \rangle$

$\vec{p} \overset{?}{\in} H$ $\quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = -1+0+2-0+2+1 \neq 0 \Rightarrow \vec{p} \notin H$

$\vec{s} \overset{?}{\in} H$ $\quad (1,3,-2) = 1\cdot(1,1,-1) + 1\cdot(0,2,-1) \Rightarrow \vec{s} \in H$



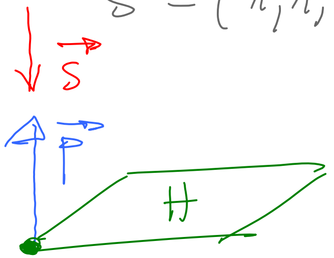
$\Rightarrow$ Sistema incompatible ("sombra infinita" o proyección)

3) $\vec{p} = \left(\frac{1}{2}, \frac{1}{2}, 1\right)$, $H = H_1$, $S = H_1^{\perp}$

$H = H_1 = \left\{ \begin{vmatrix} x & y & z \\ 1 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = 0 \right\} = \{-x+2z+2x+y = 0\} = \{x+y+2z = 0\}$

$\vec{p} \overset{?}{\in} H$: $\frac{1}{2} + \frac{1}{2} + 2 \neq 0 \Rightarrow \vec{p} \notin H$

$\vec{s} = (1,1,2)$, $\vec{s} \overset{?}{\parallel} \vec{p}$ $\quad \vec{p} = \frac{1}{2}\vec{s}$

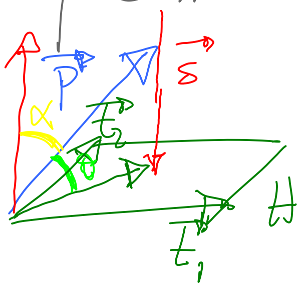


$\Rightarrow \overrightarrow{\text{Proy}_H^{\perp}(\vec{p})} = \boxed{(0,0,0)}$

4) $\vec{p} = (1,1,1)$, $H = H_2$, $S = H_2^{\perp}$

$H = H_2 = \{x - 2y - z = 0\}$, $H_2^{\perp} = \langle \underbrace{(1,-2,-1)}_{\vec{s}} \rangle$

$\vec{p} \overset{?}{\in} H$: $1 - 2 - 1 \neq 0 \Rightarrow \vec{p} \notin H$, $\vec{p} \overset{?}{\parallel} \vec{s}$ no



$$\vec{p} = \overrightarrow{\text{Proy}_H^{\perp}(\vec{p})} + \underbrace{\overrightarrow{d^{\perp}(\vec{p},H)}}_{\beta \vec{s}} = \alpha_1 \vec{t}_1 + \alpha_2 \vec{t}_2 + \beta \vec{s}$$

Como $\overrightarrow{\text{Proy}_H^{\perp}(\vec{p})} \cdot \vec{s} = 0$ multiplico por $\vec{s} \Rightarrow$

$$\vec{p} \cdot \vec{s} = \cancel{\overrightarrow{\text{Proy}} \cdot \vec{s}}^{\,0} + \beta\, \vec{s} \cdot \vec{s} \Rightarrow \beta = \frac{\vec{p} \cdot \vec{s}}{\vec{s} \cdot \vec{s}} = \frac{(1,1,1)\cdot(1,-2,-1)}{(1,-2,-1)\cdot(1,-2,-1)} = \frac{-2}{6}$$

$$\overrightarrow{\mathrm{Proy}^{\perp}_{H}(\vec{p})} = \vec{p} - \beta\vec{s} = (1,1,1) + \frac{1}{3}(1,-2,-1) = \boxed{\left(\frac{4}{3}, \frac{1}{3}, \frac{2}{3}\right)}$$

$$\vec{p}\cdot\vec{s} = \|\vec{p}\|\cdot\|\vec{s}\|\cdot\cos\alpha \Rightarrow \cos\alpha = \frac{\vec{p}\cdot\vec{s}}{\|\vec{p}\|\cdot\|\vec{s}\|}, \quad \theta = 90^\circ - \alpha$$

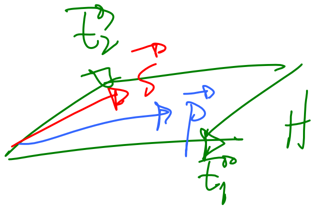
Distancia de $\vec{p}$ a H $= \|\overrightarrow{d^{\perp}(\vec{p},H)}\| = \|\beta\vec{s}\| = \frac{1}{3}\|(1,-2,-1)\| = \frac{\sqrt{6}}{3}$

5) $\vec{p} = \left(1, \frac{1}{3}, \frac{1}{3}\right)$, $H = H_2$, $S = H_4 = \begin{cases} y - z = 0 \\ 2x - 3y - 3z = 0 \end{cases} = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 2 & -3 & -3 \end{vmatrix} \right\rangle$

$= \langle \underbrace{(3,1,1)}_{\vec{s}} \rangle$

$H_2 = \{x - 2y - z = 0\} = H$

$\vec{p} \underset{si}{\overset{?}{\in}} H : 1 - \frac{2}{3} - \frac{1}{3} = 0$, $\vec{s} \underset{si}{\overset{?}{\in}} H : 3 - 2 - 1 = 0$



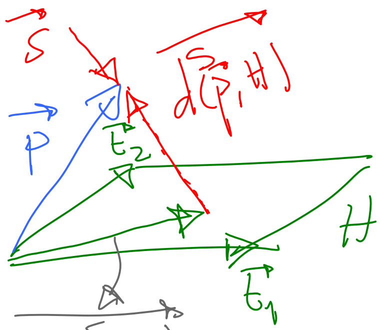
Sistema Compatible indeterminado = infinitas soluciones

$\boxed{S.C.I.}$

6) $\vec{p}=(1,1,1)$, $H=H_1$, $S=H_4=\langle(3,1,1)\rangle$

$$H=H_1=\langle \underbrace{(1,1,-1)}_{\vec t_1}, \underbrace{(0,2,-1)}_{\vec t_2}\rangle = \{x+y+2z=0\}$$

$\vec p \overset{?}{\in} H$: $1+1+2\neq 0$, no ; $\vec s \overset{?}{\in} H$: $3+1+2\neq 0$, no ; $\vec p \parallel \vec s$ no



$\overrightarrow{Proy_H^S(\vec p)}$

$$\vec p = \overrightarrow{Proy_H^S(\vec p)} + \underbrace{\overrightarrow{d(\vec p,H)}}_{\beta\vec s} = \underbrace{\alpha_1\vec t_1+\alpha_2\vec t_2}_{(x,y,z)}+\beta\vec s$$

$$\left.\begin{array}{l}(1,1,1)=(x,y,z)+\beta(3,1,1)\\ \overrightarrow{Proy_H^S(\vec p)}=(x,y,z)\in H \Rightarrow x+y+2z=0\end{array}\right\}$$

$$(x,y,z)=(1,1,1)-(3\beta,\beta,\beta)=(\underbrace{1-3\beta}_{x},\underbrace{1-\beta}_{y},\underbrace{1-\beta}_{z})\Rightarrow$$

$$(1-3\beta)+(1-\beta)+2(1-\beta)=0 \Rightarrow 4-6\beta=0 \Rightarrow \beta=\frac{2}{3}$$

$$\overrightarrow{\text{Proy}_H^S}(\vec{p}) = (x,y,z) = (1-3\beta, 1-\beta, 1-\beta) = \left(1-2, 1-\frac{2}{3}, 1-\frac{2}{3}\right) =$$

$$= \boxed{\left(-1, \frac{1}{3}, \frac{1}{3}\right)}$$

BASE ORTONORMAL DE UN SUBESPACIO VECTORIAL $H \subset \mathbb{R}^n$

Def) Dada una base B de $H \subset \mathbb{R}^n$ se dice que B es ortonormal si sus vectores son perpendiculares dos a dos y tienen módulo 1.

Ej) Calcula una base ortonormal de $H = \{x+2y+z=0\}$

$H = \langle \underbrace{(1,-1,1)}_{\vec{t}_1}, \underbrace{(2,-1,0)}_{\vec{t}_2} \rangle$, $\vec{t}_1 \cdot \vec{t}_2 = 2+1+0 = 3 \neq 0$

$$\hat{t}_1 = \frac{\vec{t}_1}{\|\vec{t}_1\|} = \frac{(1,-1,1)}{\sqrt{3}}$$

$$\boxed{\vec{t}'_2} = \vec{t}_2 - (\vec{t}_2 \cdot \hat{t}_1)\hat{t}_1 =$$

$$\mathrm{Proy}_{\vec{t}_1}(\vec{t}_2) = (\vec{t}_2 \cdot \hat{t}_1)\,\hat{t}_1$$

$$= (2,-1,0) - \frac{(2,-1,0)\cdot(1,-1,1)}{\sqrt{3}}\,\frac{(1,-1,1)}{\sqrt{3}} =$$

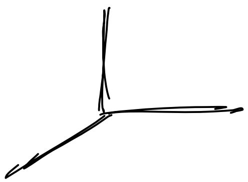
$$= (2,-1,0) - \frac{3}{3}(1,-1,1) = \boxed{(1,0,-1)}$$

Comprobación $\vec{t}'_2 \cdot \vec{t}_1 = 1 - 1 = 0$

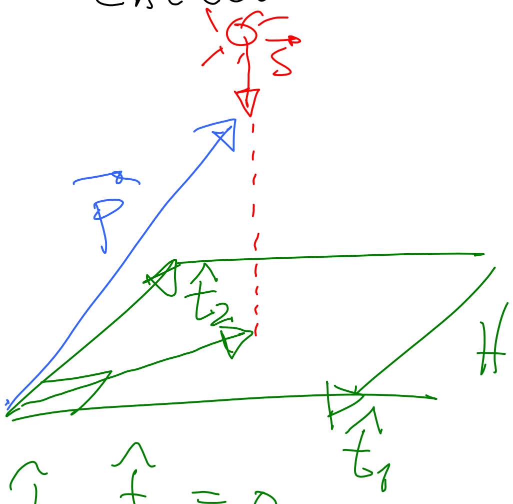
$$\hat{t}'_2 = \frac{\vec{t}'_2}{\|\vec{t}'_2\|} = \frac{(1,0,-1)}{\sqrt{2}} \Longrightarrow \boxed{B_{\text{ortonormal}}(H) = \left\{\frac{(1,-1,1)}{\sqrt{3}}, \frac{(1,0,-1)}{\sqrt{2}}\right\}}$$

Otra forma:

$$\left.\begin{array}{l} \vec{t}'_2 = \vec{t}_2 + \alpha\,\vec{t}_1 \\ \vec{t}'_2 \cdot \vec{t}_1 = 0 \end{array}\right\} \xrightarrow{\cdot\,\vec{t}_1} 0 = \vec{t}_2 \cdot \vec{t}_1 + \alpha\,\vec{t}_1 \cdot \vec{t}_1 \Longrightarrow \alpha = \frac{\vec{t}_2 \cdot \vec{t}_1}{\|\vec{t}_1\|^2}$$



CALCULO DE PROYECCIONES ORTOGONALES USANDO BASES ORTONORMALES



$\hat{t}_1 \cdot \hat{t}_2 = 0$

$\|\hat{t}_1\| = 1 = \|\hat{t}_2\|$

$$\vec{p} = \underbrace{\alpha_1 \hat{t}_1 + \alpha_2 \hat{t}_2}_{\overrightarrow{\mathrm{Proy}^{\perp}_H(\vec{p})}} + \underbrace{\beta \vec{s}}_{\overrightarrow{d^{\perp}(\vec{p}, H)}}$$

$$\vec{p} \cdot \hat{t}_1 = \alpha_1 \underbrace{\hat{t}_1 \cdot \hat{t}_1}_{1} + \alpha_2 \cancel{\hat{t}_2 \cdot \hat{t}_1}^{0} + \beta \cancel{\vec{s} \cdot \hat{t}_1}^{0} \Rightarrow \boxed{\alpha_1 = \vec{p} \cdot \hat{t}_1}$$

$$\vec{p} \cdot \hat{t}_2 = \alpha_1 \cancel{\hat{t}_1 \cdot \hat{t}_2}^{0} + \alpha_2 \underbrace{\hat{t}_2 \cdot \hat{t}_2}_{1} + \beta \cancel{\vec{s} \cdot \hat{t}_2}^{0} \Rightarrow \boxed{\alpha_2 = \vec{p} \cdot \hat{t}_2}$$

$$\vec{p} \cdot \vec{s} = \alpha_1 \cancel{\hat{t}_1 \cdot \vec{s}}^{0} + \alpha_2 \cancel{\hat{t}_2 \cdot \vec{s}}^{0} + \beta \vec{s} \cdot \vec{s} \Rightarrow \boxed{\beta = \frac{\vec{p} \cdot \vec{s}}{\vec{s} \cdot \vec{s}}}$$

Ej] Calcular $\overrightarrow{\mathrm{Proy}^{\perp}_H(\vec{p})}$, donde $H = \{x + 2y + z = 0\}$ y $\vec{p} = (1,1,1)$

$$H = \langle \hat{t}_1, \hat{t}_2 \rangle = \left\langle \frac{(1,-1,1)}{\sqrt{3}}, \frac{(1,0,-1)}{\sqrt{2}} \right\rangle$$

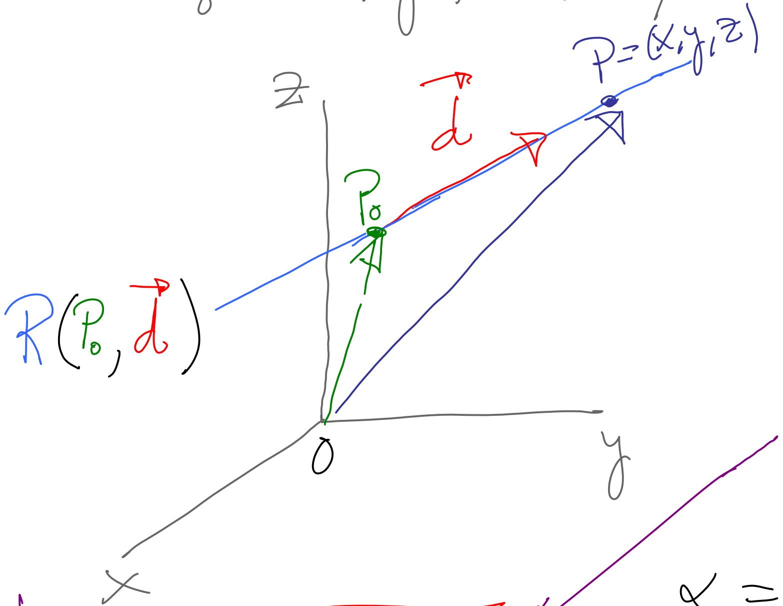
$$\alpha_1 = \vec{p} \cdot \hat{t}_1 = (1,1,1) \cdot \frac{(1,-1,1)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha_2 = \vec{p} \cdot \hat{t}_2 = (1,1,1) \cdot \frac{(1,0,-1)}{\sqrt{2}} = 0$$

$$\mathrm{Proy}_H^{\perp}(\vec{p}) = \alpha_1 \hat{t}_1 + \alpha_2 \hat{t}_2 = \frac{1}{\sqrt{3}} \frac{(1,-1,1)}{\sqrt{3}} + \vec{0} = \boxed{\frac{(1,-1,1)}{3}}$$

ESPACIO AFIN

A) ECUACIONES DE UNA RECTA QUE PASA POR UN PUNTO $P_0=(x_0,y_0,z_0)$ Y TIENE VECTOR DIRECTOR $\vec{d}=(a,b,c)$



$$\vec{OP} = \vec{OP_0} + \alpha\,\vec{d}$$

$$(x,y,z) = (x_0,y_0,z_0) + \alpha(a,b,c)$$ Ec. paramétricas

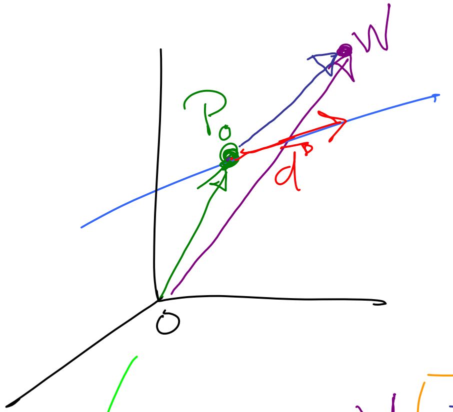
$$(\overbrace{x-x_0}^{x'},\ \overbrace{y-y_0}^{y'},\ \overbrace{z-z_0}^{z'}) = \alpha(a,b,c)$$

$$\alpha = \boxed{\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}}$$ Forma Continua

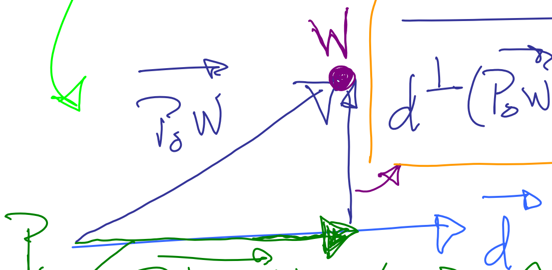
Otra forma:

$$\text{Rango}\begin{pmatrix} x' & y' & z' \\ a & b & c \end{pmatrix} = 1 \Rightarrow \left\{\begin{array}{l} \begin{vmatrix} x' & y' \\ a & b \end{vmatrix} = 0 \Rightarrow \boxed{x'b - y'a = 0} \\ \begin{vmatrix} y' & z' \\ b & c \end{vmatrix} = 0 \Rightarrow \boxed{y'c - z'b = 0} \end{array}\right\}$$

Ej) Calcula la distancia de un pto $W = (w_1, w_2, w_3) = \overrightarrow{OW}$ a la recta anterior.



$$\overrightarrow{OW} = \overrightarrow{OP_0} + \overrightarrow{P_0W} =$$
$$= (x_0, y_0, z_0) + \overrightarrow{P_0W}$$
$$\overrightarrow{P_0W} = \overrightarrow{OW} - \overrightarrow{OP_0} = (w_1, w_2, w_3) - (x_0, y_0, z_0)$$

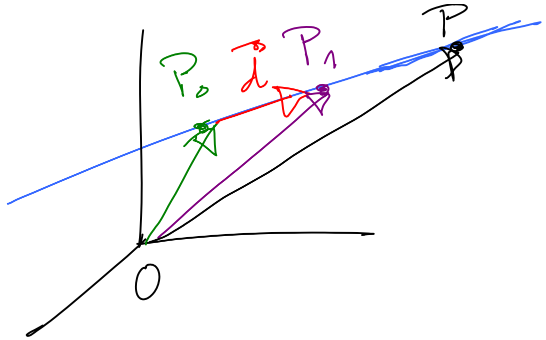


$$\boxed{d^{\perp}(\overrightarrow{P_0W}, R) = \overrightarrow{P_0W} - (\overrightarrow{P_0W} \cdot \hat{d})\,\hat{d}}$$

$$\text{Proy}^{\perp}_{\vec{d}}(\overrightarrow{P_0W}) = (\overrightarrow{P_0W} \cdot \hat{d})\,\hat{d}$$

A.1) ECUACIÓN DE LA RECTA QUE PASA POR DOS PUNTOS: P_0 y P_1

$$\overrightarrow{OP_0} + \vec{d} = \overrightarrow{OP_1}$$



$\vec{d} = \overrightarrow{OP_1} - \overrightarrow{OP_0} = \overrightarrow{P_0P_1}$

Este caso se reduce al anterior

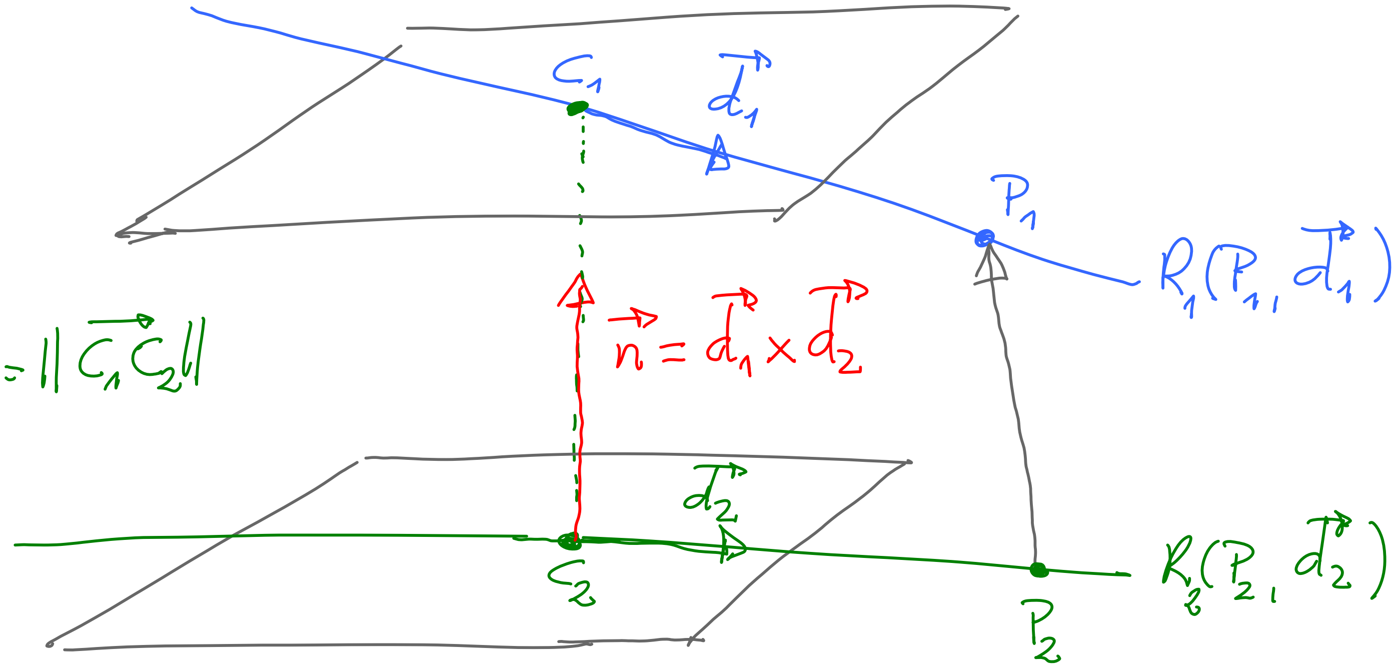
Ej] Calcula la distancia más corta entre dos rectas:

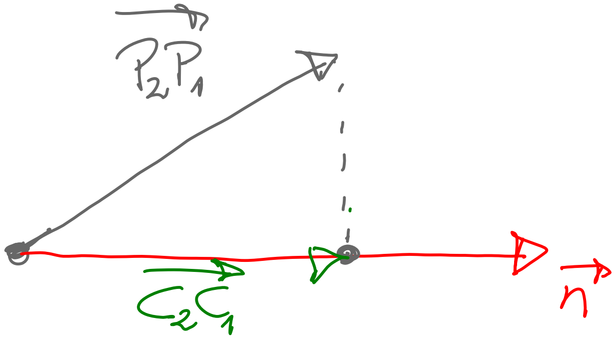
$R_1 = (P_1, \vec{d}_1)$ y $R_2 = (P_2, \vec{d}_2)$.

[Este ejercicio lo volveremos a hacer en el Bloque III por el procedimiento de "mínimos cuadrados".]

[Véase también la página web: http://www.ciencialab.com/mod/resource/view.php?id=54]

$$d(R_1, R_2) = \|\overrightarrow{C_1C_2}\|$$

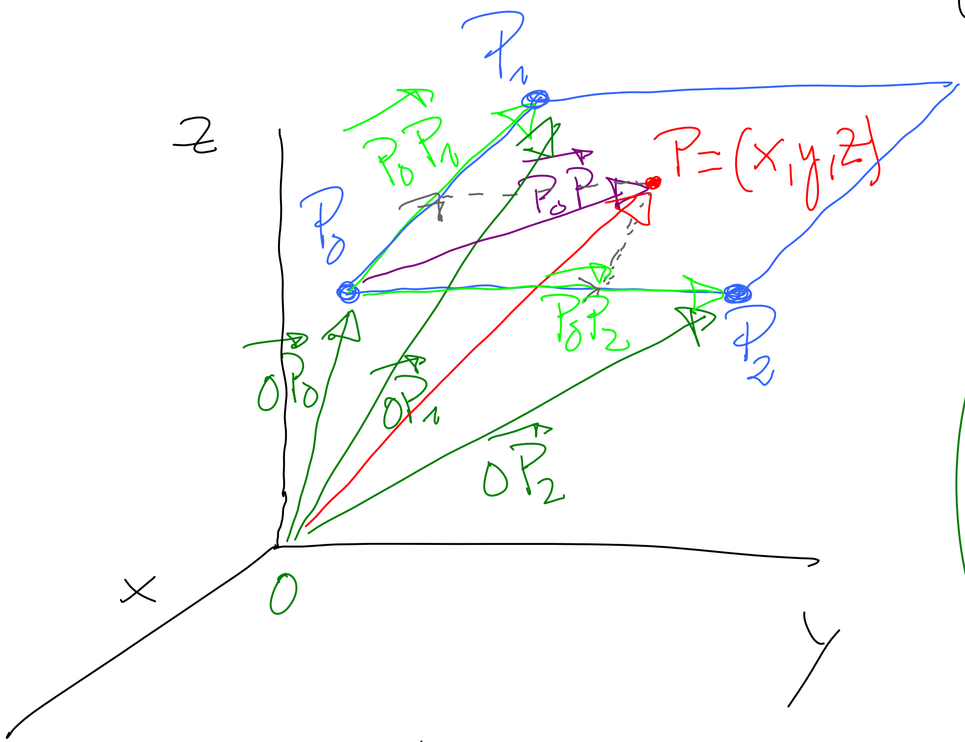




$$\overrightarrow{C_2C_1} = (\overrightarrow{P_2P_1} \cdot \hat{n})\,\hat{n}$$

$$d(R_1, P_2) = \|\overrightarrow{C_2C_1}\| = |\overrightarrow{P_2P_1} \cdot \hat{n}| = \frac{|\overrightarrow{P_2P_1} \cdot \vec{d_1} \times \vec{d_2}|}{\|\vec{d_1} \times \vec{d_2}\|}$$

B] ECUACIONES DE UN PLANO QUE PASA POR TRES PUNTOS NO COLINEALES $P_0=(x_0,y_0,z_0)$, $P_1=(x_1,y_1,z_1)$, $P_2=(x_2,y_2,z_2)$



$$\vec{P_0P} = \alpha\,\vec{P_0P_1} + \beta\,\vec{P_0P_2}$$

$$\vec{P_0P} = \vec{OP} - \vec{OP_0} = (\underbrace{x-x_0}_{x'}, \underbrace{y-y_0}_{y'}, \underbrace{z-z_0}_{z'})$$

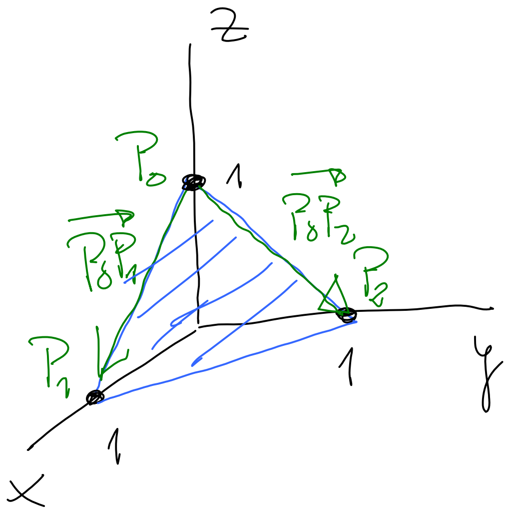
$$\vec{P_0P_1} = \vec{OP_1} - \vec{OP_0} = (\underbrace{x_1-x_0}_{x_1'}, \underbrace{y_1-y_0}_{y_1'}, \underbrace{z_1-z_0}_{z_1'})$$

$$\vec{P_0P_2} = \vec{OP_2} - \vec{OP_0} = (\underbrace{x_2-x_0}_{x_2'}, \underbrace{y_2-y_0}_{y_2'}, \underbrace{z_2-z_0}_{z_2'})$$

$$(x',y',z') = \alpha(x_1',y_1',z_1') + \beta(x_2',y_2',z_2')$$

$$\begin{vmatrix} x' & y' & z' \\ x_1' & y_1' & z_1' \\ x_2' & y_2' & z_2' \end{vmatrix} = 0 \longleftarrow \text{Ecuaciones implícitas del plano}$$

Ej] Calcula las ecuaciones implícitas del plano que pasa por los siguientes tres puntos $P_0=(0,0,1)$, $P_1=(1,0,0)$, $P_2=(0,1,0)$



$$\vec{P_0P} = \vec{OP} - \vec{OP_0} = (x-0,\ y-0,\ z-1)$$

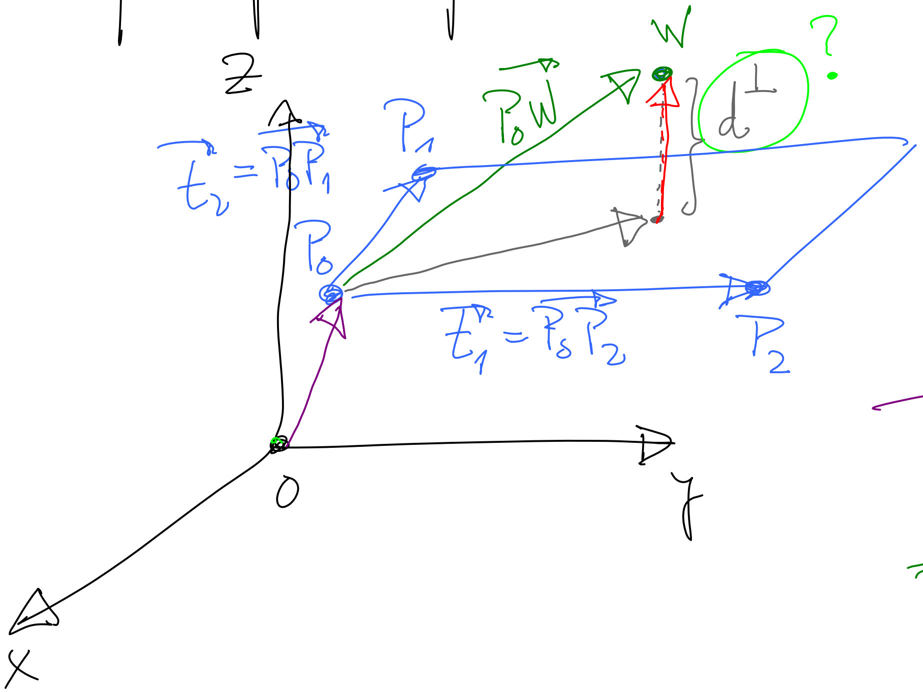
$$\vec{P_0P_1} = \vec{OP_1} - \vec{OP_0} = (1-0,\ 0-0,\ 0-1)$$

$$\vec{P_0P_2} = \vec{OP_2} - \vec{OP_0} = (0-0,\ 1-0,\ 0-1)$$

$$\begin{vmatrix} x & y & (z-1) \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = 0 \Rightarrow (z-1)+x+y=0$$

$$\boxed{x+y+z=1}$$

Ej] Calcula la distancia de $W=(w_1, w_2, w_3)$ al plano que pasa por $P_0=(x_0, y_0, z_0)$, $P_1=(x_1, y_1, z_1)$, $P_2=(x_2, y_2, z_2)$



$$\vec{p}=\vec{P_0W}=\vec{OW}-\vec{OP_0}=(w_1-x_0, w_2-y_0, w_3-z_0)$$

$$\vec{s}=\vec{t_1}\times\vec{t_2}=\vec{P_0P_2}\times\vec{P_0P_1}$$

Todo se reduce al caso estudiado: