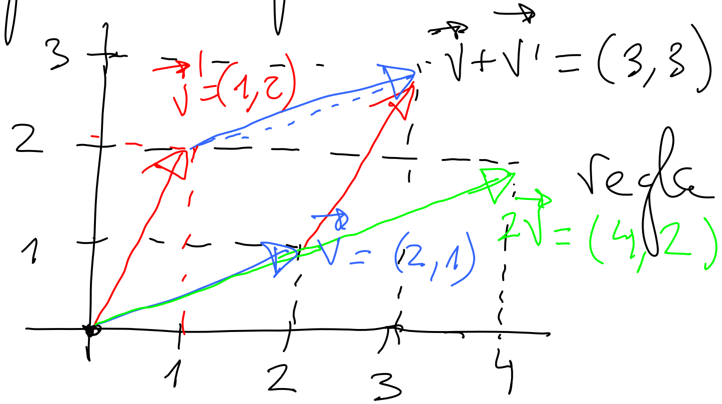
El ESPACIO $\mathbb{R}^n$

$$\mathbb{R}^n = \{ \vec{v} = \underbrace{(v_1, v_2, \ldots, v_n)}_{\text{Vector}},\ v_i \in \mathbb{R} \}$$

Operaciones:

E.V $\begin{cases} 1)\ \text{Suma} \quad \vec{v} + \vec{v}' = (v_1 + v_1', v_2 + v_2', \ldots, v_n + v_n') \\ 2)\ \text{Multiplicación por escalares} \quad \alpha \vec{v} = (\alpha v_1, \alpha v_2, \ldots, \alpha v_n) \end{cases}$

$\mathbb{R}^2$



regla del paralelogramo

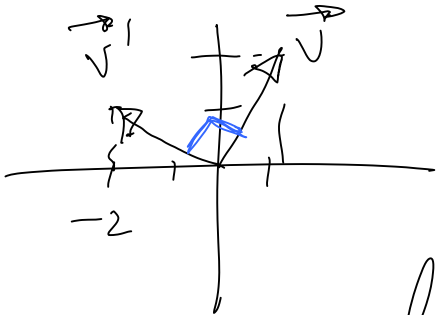
3) Producto escalar $\quad \vec{v} \cdot \vec{v}' = v_1 \cdot v_1' + v_2 \cdot v_2' + \cdots + v_n v_n' \in \mathbb{R}$

$\langle \vec{v}, \vec{v}' \rangle$

Se dice que $\vec{v} \perp \vec{v}' \iff \vec{v} \cdot \vec{v}' = 0 \quad , \vec{v}, \vec{v}' \neq 0$

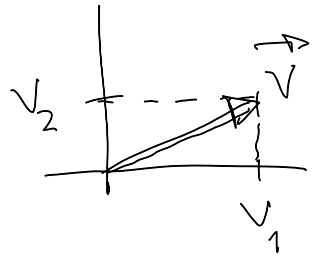
"es perpendicular"

$\vec{v} = (1,2) \perp \vec{v}' = (-2,1) \qquad (1,2) \cdot (-2,1) = -2 + 2 = 0$



Norma o longitud de un vector $\quad \|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$



Vector unitario en la dirección de $\vec{v}$: $\hat{v} = \dfrac{\vec{v}}{\|\vec{v}\|}$

$\|\hat{v}\| = 1$

4) Producto vectorial en $\mathbb{R}^3$ $\hat{\imath} = (1,0,0)$, $\hat{\jmath} = (0,1,0)$, $\hat{k} = (0,0,1)$

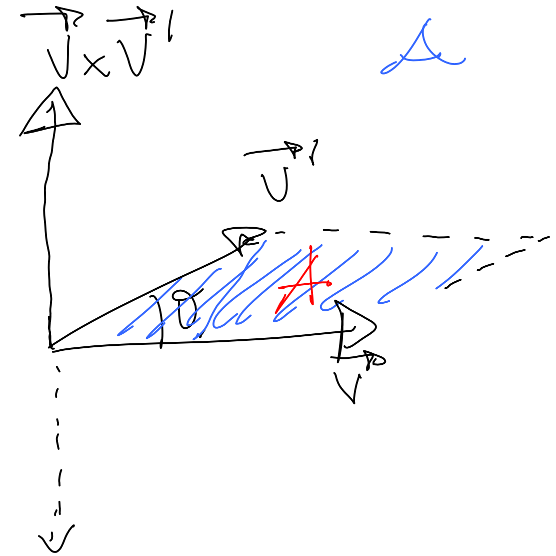
$$\vec{v} \times \vec{v}' = \text{"}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ v_1 & v_2 & v_3 \\ v_1' & v_2' & v_3' \end{vmatrix}\text{"} = \hat{\imath}\begin{vmatrix} v_2 & v_3 \\ v_2' & v_3' \end{vmatrix} - \hat{\jmath}\begin{vmatrix} v_1 & v_3 \\ v_1' & v_3' \end{vmatrix} + \hat{k}\begin{vmatrix} v_1 & v_2 \\ v_1' & v_2' \end{vmatrix}$$

$$= \left(\begin{vmatrix} v_2 & v_3 \\ v_2' & v_3' \end{vmatrix}, \begin{vmatrix} v_3 & v_1 \\ v_3' & v_1' \end{vmatrix}, \begin{vmatrix} v_1 & v_2 \\ v_1' & v_2' \end{vmatrix} \right)$$

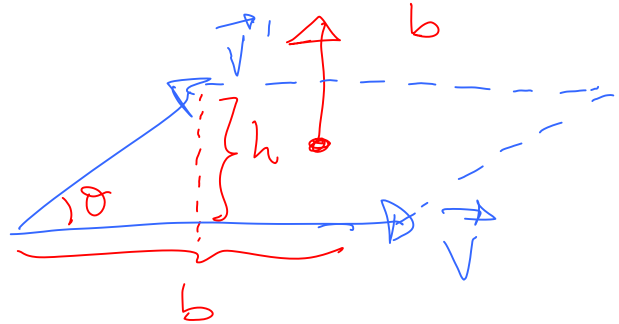
$$\boxed{\vec{v} \cdot (\vec{v} \times \vec{v}')} = v_1 \begin{vmatrix} v_2 & v_3 \\ v_2' & v_3' \end{vmatrix} + v_2 \begin{vmatrix} v_3 & v_1 \\ v_3' & v_1' \end{vmatrix} + v_3 \begin{vmatrix} v_1 & v_2 \\ v_1' & v_2' \end{vmatrix} =$$

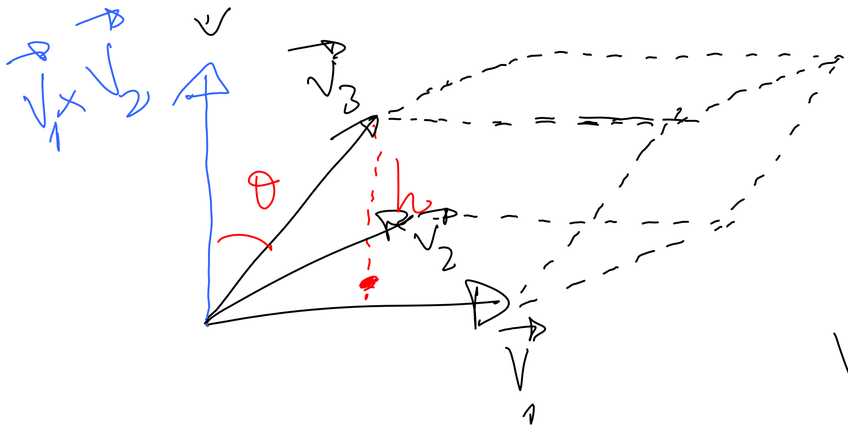
$$= \begin{vmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1' & v_2' & v_3' \end{vmatrix} \underset{F_1 \to F_1 - F_2}{=} \begin{vmatrix} 0 & 0 & 0 \\ v_1 & v_2 & v_3 \\ v_1' & v_2' & v_3' \end{vmatrix} = \boxed{0}$$

$$\boxed{\vec{v}' \cdot (\vec{v} \times \vec{v}') = \cdots = 0}$$



$$\|\vec{v}\times\vec{v}'\| = \underbrace{\|\vec{v}\|}_{b} \cdot \underbrace{\|\vec{v}'\| \cdot \operatorname{sen}\theta}_{h} = A$$

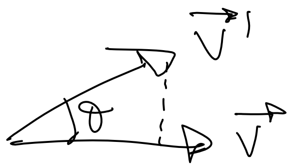




$$\vec{v}_3 \cdot (\vec{v}_1 \times \vec{v}_2) = \begin{vmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{vmatrix}$$

Volumen del paralelepipedo

$$\vec{v}\cdot\vec{v}' = \|\vec{v}\| \cdot \|\vec{v}'\| \cdot \cos\theta$$



DEPENDENCIA E INDEPENDENCIA LINEAL

Def) Dada una familia $F = \{\vec{v}_1, \dots, \vec{v}_m\} \subset \mathbb{R}^n$ se dice que $\vec{v} \in \mathbb{R}^n$ depende linealmente de F sii existen escalares $\alpha_1, \dots, \alpha_m \in \mathbb{R}$ tales que

$$\vec{v} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots + \alpha_m \vec{v}_m$$

Ej) $F = \{\vec{v}_1 = (1,2)\}$ $\quad \underline{\vec{v} = (1,1)} = \alpha \vec{v}_1 = \underline{\alpha(1,2)}$

$$\Rightarrow \left.\begin{array}{l} 1 = \alpha \\ 1 = 2\alpha \end{array}\right\} \Rightarrow \cancel{\begin{array}{l} \alpha = 1 \\ \alpha = 1/2 \end{array}}$$

$\vec{v}$ no depende ~~lin~~ de F

Ej) $F = \{\vec{v}_1 = (1,2), \vec{v}_2 = (1,1)\}$

$\vec{v} = (2,3) = \alpha(1,2) + \beta(1,1) \quad \alpha = 1, \ \beta = 1$

$\vec{v}$ depende de F

Def) Se dice que F es ligada cuando alguno de sus vectores depende del resto. De lo contrario se dice que F es libre o que sus vectores son linealmente independientes.

Ej) $F = \{\vec{v}_1 = (1,2), \vec{v}_2 = (1,4)\}$ <u>libre</u>

¿ $\vec{v}_2 = \alpha \vec{v}_1$? $(1,4) = \alpha(1,2) \Leftrightarrow \left.\begin{array}{l} 1 = \alpha \\ 4 = 2\alpha \end{array}\right\} \cancel{\begin{array}{l} \alpha = 1 \\ \alpha = 2 \end{array}}$

$F = \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix} \qquad |F| = 4 \cdot 1 - 2 \cdot 1 = 2 \neq 0 \Rightarrow$

$\vec{v}_1, \vec{v}_2$ son indep.

Ej) $F = \{ (1,2,3), (1,0,1), (0,2,2) \}$ ¿libre?

$$F = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{pmatrix} \qquad |F| = 0 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} =$$

$$= 4 - 4 = 0 \Rightarrow F \text{ es ligada}$$

Def) rango(F) = nº de vectores indep. de F

= orden de la submatriz cuadrada de F más grande con determinante no nulo

orden = nº filas ó col.

$\Rightarrow$ rango$(F) < 3$

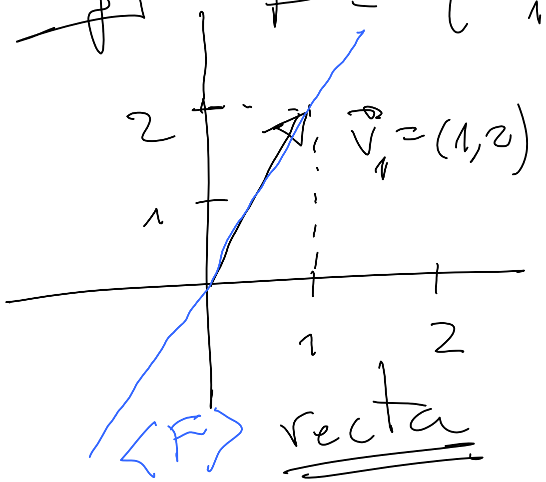
$$\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2 \neq 0 \Rightarrow \text{rango}(F) = 2$$

ENVOLTURA LINEAL O SUBESPACIO VECTORIAL ENGENDRADO POR UNA FAMILIA F DE VECTORES

Def) Dada una familia $F=\{\vec{v}_1,\dots,\vec{v}_m\}\subset\mathbb{R}^n$ se define su envoltura lineal como:

$$\langle F\rangle = \{\vec{v} = \alpha_1\vec{v}_1+\alpha_2\vec{v}_2+\dots+\alpha_m\vec{v}_m\}$$

Ej) $F=\{\vec{v}_1=(1,2)\}\subset\mathbb{R}^2$

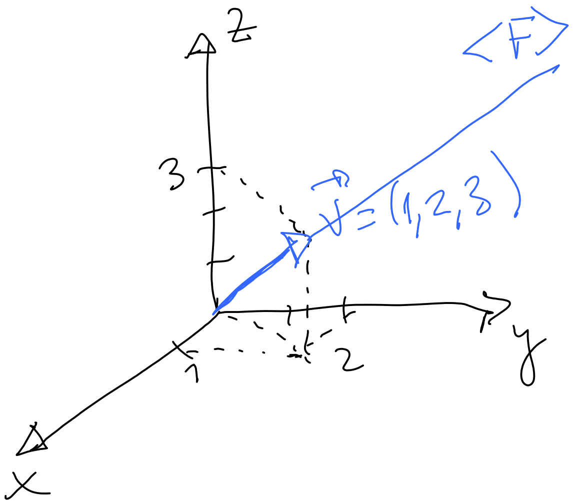


$$\langle F\rangle=\{\vec{v}=\alpha\vec{v}_1\}=\{(x,y)=\alpha(1,2)\}$$

Parámetro libre o grado de libertad

$$= \underbrace{\left\{\begin{array}{l}x=\alpha\cdot 1\\ y=\alpha\cdot 2\end{array}\right\}}_{\text{Ecuaciones paramétricas}} \overset{\alpha=x}{=} \underbrace{\{y=2x\}}_{\text{Ecuaciones implícitas o ligaduras}}$$

Ej. $F = \{(1,2,3)\} \subset \mathbb{R}^3$



$$\langle F\rangle = \{\vec{v} = \alpha(1,2,3) = (x,y,z)\} =$$

1 grado de libertad

$$= \left\{\begin{array}{l} x = \alpha \\ y = 2\alpha \\ z = 3\alpha \end{array}\right\} \overset{\alpha = x}{=} \left\{ \boxed{\begin{array}{l} y = 2x \\ z = 3x \end{array}} \right\}$$

Ec. paramétricas

Ec. implícitas ó ligaduras

$\dim(\mathbb{R}^3) =$ nº de grados de libertad $(\mathbb{R}^3) = 3$

$$\boxed{\text{nº de grados de libertad}(F) = \dim\langle F\rangle = \dim(\mathbb{R}^3) - \begin{pmatrix}\text{nº de} \\ \text{ligaduras}\end{pmatrix}}$$

$$1 = \text{rango}(F) = 3 - 2$$

otra forma de obtener las ecuaciones implícitas:

$$\text{rango}\begin{pmatrix} x & y & z \\ 1 & 2 & 3 \end{pmatrix} = 1 \Rightarrow \left\{ \begin{array}{l} \begin{vmatrix} x & y \\ 1 & 2 \end{vmatrix} = 0 \Leftrightarrow \boxed{2x - y = 0} \\ \begin{vmatrix} y & z \\ 2 & 3 \end{vmatrix} = 0 \Leftrightarrow \cancel{3y - 2z = 0} \end{array} \right\}$$

$$\begin{vmatrix} x & z \\ 1 & 3 \end{vmatrix} = 0 \Rightarrow \boxed{3x - z = 0}$$

Ej

$$F = \{ (1,2,3), (1,0,1) \} \subset \mathbb{R}^3$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix}$$

$$1 \cdot 0 - 2 \cdot 1 = -2 \neq 0$$

$$\text{rango}(F) = 2$$

$$\langle F \rangle = \{ (x,y,z) = \alpha(1,2,3) + \beta(1,0,1) \} =$$

$$= \underbrace{\left\{ \begin{array}{l} x = \alpha + \beta \\ y = 2\alpha \\ z = 3\alpha + \beta \end{array} \right\}}_{\text{Ec. param.}} = \underbrace{\{ \boxed{x + y - z = 0} \}}_{\text{Ec. implícitas ó ligaduras}}$$

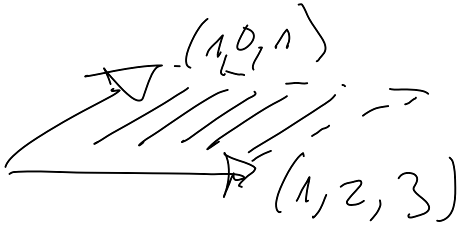
Plano de $\mathbb{R}^3$

Otra forma de obtener las ecuaciones implícitas sin necesidad de pasar por las paramétricas

$$\text{rango}\begin{pmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} = 2 \Rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{vmatrix} = 0 \quad \text{Sarrus}$$

$$2x + 3y + 0 \cdot z - (2z + 0 \cdot x + y) = 0 \Rightarrow 2x + 2y - 2z = 0$$

dividido por 2 $\Rightarrow$ $\boxed{x + y - z = 0}$



$$\underbrace{\text{rango}(F)}_{2} = \underbrace{\dim(\mathbb{R}^3)}_{3} - \underbrace{\text{nº de ligad.}}_{1}$$

Def] Dado un subespacio vectorial $H \subset \mathbb{R}^n$ se dice que la familia $F \subset \mathbb{R}^n$ es una base de H sii $\langle F \rangle = H$ y F es libre.

Ej] Dado $H = \{(x,y,z) \in \mathbb{R}^3 : x - 2y + z = 0\}$ ← plano

Calcula una base de H

$$\left.\begin{array}{l} x = \alpha \\ y = \beta \end{array}\right\} z = 2y - x = 2\beta - \alpha$$

$\alpha = 0,\ \beta = 1$: $(x,y,z) = (0,1,2) \in H$

$\alpha = -1,\ \beta = 0$: $(x,y,z) = (-1,0,1) \in H$

$$F = \begin{pmatrix} \boxed{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}} & \begin{matrix} 2 \\ 1 \end{matrix} \end{pmatrix} \rightarrow \text{rango}(F) = 2 \Rightarrow F \text{ libre} \Rightarrow F \text{ es base}$$

$1 \neq 0$

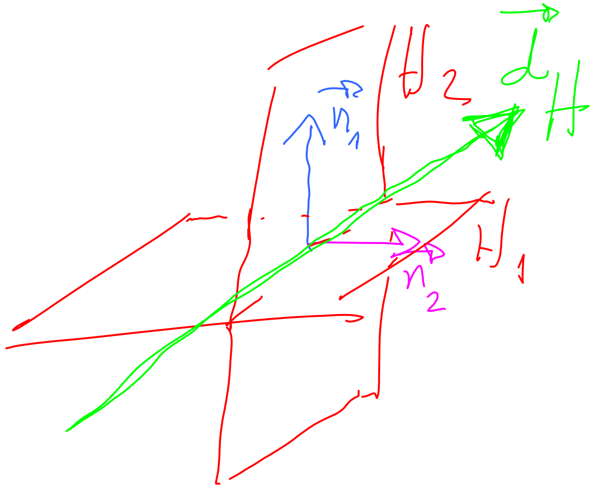
Ej) Dados $H = \{(x,y,z) \in \mathbb{R}^3 : \left.\begin{matrix} \underbrace{x+y-z=0}_{H_1} \\ \underbrace{2x-y+3z=0}_{H_2} \end{matrix}\right\}$ ← recta

Calcula una base F

1) <u>1ª forma</u> $x = \alpha$

$$\left.\begin{matrix} y - z = -\alpha \\ -y+3z = -2\alpha \end{matrix}\right\} \begin{matrix} \xrightarrow{\alpha = 0} \ ¿\cancel{(0,0,0)}? \ ! \\ \xrightarrow{\hspace{1.5em}} (2,-5,-3) \\ \alpha = 2 \end{matrix}$$

2) <u>2ª forma</u>



$$\vec{n}_1 = (1, 1, -1)$$

$$\vec{n}_2 = (2, -1, 3)$$

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$\vec{d} = \left(\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \right) = (2,-5,-3) \overset{?}{\in} H \text{ ¡}\underline{\underline{\text{si}}}\text{!}$$

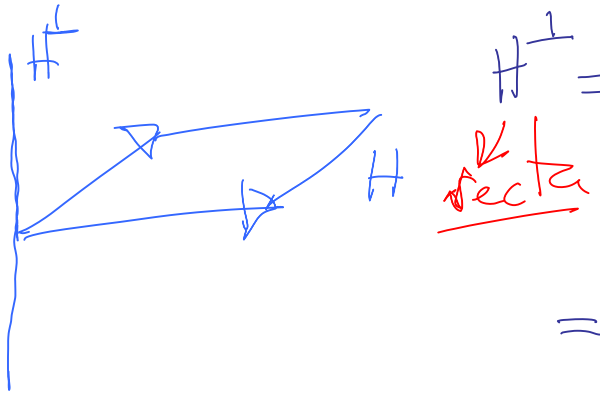
$F = \{(2,-5,-3)\}$ es una base de H.

Def Dado un subespacio vectorial $H \subset \mathbb{R}^n$ se define el subespacio ortogonal a H como:

$$H^{\perp} = \{ \vec{v} \in \mathbb{R}^n : \vec{v} \cdot \vec{h} = 0 \quad \forall \vec{h} \in H \}$$

Ej Dado $H = \{(x,y,z) \in \mathbb{R}^3 : x - 4y + 5z = 0\}$ plano

Calcula $H^{\perp}$. (ec. implícitas y base)



$$H^\perp = \left\{ (a,b,c)\in\mathbb{R}^3 : \underbrace{(a,b,c)\cdot(x,y,z)}_{ax+by+cz=0} = 0 \quad \forall (x,y,z)\in H \right\}$$

$$\underset{1}{a}x + \underset{-4}{b}y + \underset{5}{c}z = 0$$

base de $H^\perp$

$$= \langle (1,-4,5) \rangle = \begin{cases} -4x - y = 0 \\ 5y + 4z = 0 \end{cases}$$

$$\text{rango} \begin{pmatrix} x & y & z \\ 1 & -4 & 5 \end{pmatrix} = 1$$

$$\begin{vmatrix} x & y \\ 1 & -4 \end{vmatrix} = 0 \Rightarrow -4x - y = 0$$

$$\begin{vmatrix} y & z \\ -4 & 5 \end{vmatrix} = 0 \Rightarrow 5y + 4z = 0$$

$$\begin{vmatrix} x & z \\ 1 & 5 \end{vmatrix} = 0 \Rightarrow 5x - z = 0$$ combinación de

$\boxed{\text{Teorema}}$

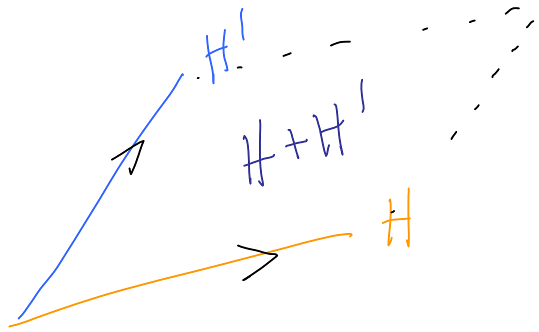
$$\dim(H) + \dim(H^\perp) = \dim(\mathbb{R}^n) = n$$

$$\mathbb{R}^n = H \oplus H^\perp$$

Def Dados dos subespacios $H, H' \subset \mathbb{R}^n$ se define su suma como:

$$H + H' = \{ \vec{v} \in \mathbb{R}^n : \vec{v} = \vec{h} + \vec{h}', \ \vec{h} \in H, \ \vec{h}' \in H' \}$$

Ej $H = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} x+y=0 \\ z=0 \end{array} \right\}$ recta , $H' = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} x-y=0 \\ y+z=0 \end{array} \right\}$ recta



$$H = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \right\rangle = \langle (1,-1,0) \rangle$$

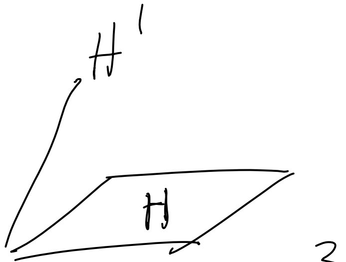
$$H' = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \right\rangle = \langle (1,1,-1) \rangle$$

$$H+H' = \langle (1,-1,0),(1,1,-1) \rangle = \left\{ \underbrace{\begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix} = 0}_{x+y+2z=0} \right\}$$

plano

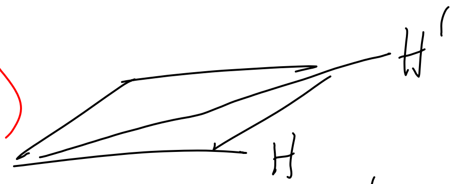
Ej:

a)



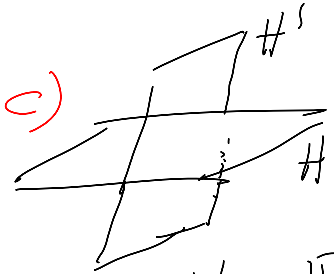
$H+H' = \mathbb{R}^3$

b)



$H+H' = H$

c)



$H+H' = \mathbb{R}^3$

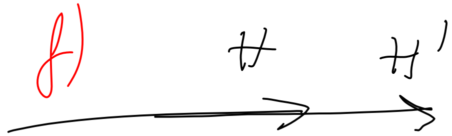
d)

$H+H' = H = H'$

e)

$H+H' = \mathbb{R}^2$

f)

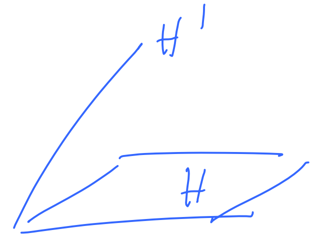


$H+H' = H = H'$

Ej a)

$$H = \{(x,y,z) \in \mathbb{R}^3 : x + 2y = 0\}$$

$$H' = \langle (1,0,0) \rangle$$



¿ $H' \subset H$? no porque $(1,0,0)$ no verifica la ecuación $x+2y=0$

$$H = \langle (0,0,1), (2,-1,0) \rangle$$

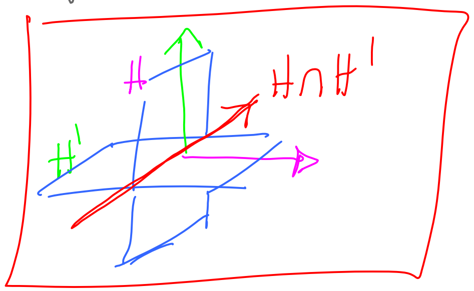
$$H + H' = \langle \underbrace{(0,0,1), (2,-1,0)}_{\in H}, \underbrace{(1,0,0)}_{\in H'} \rangle$$

$$\begin{vmatrix} 0 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \neq 0 \Rightarrow \operatorname{rang}(H + H') = 3 \Rightarrow H + H' = \mathbb{R}^3$$

Def) Dados dos subespacios vect. $H, H' \subset \mathbb{R}^n$ se define su intersección como:

$$H \cap H' = \{ \vec{v} \in \mathbb{R}^n : \vec{v} \in H, \vec{v} \in H' \}$$

Ej) $H = \{(x,y,z) \in \mathbb{R}^3 : x+y=0\}$, $H' = \langle \underbrace{(1,2,1)}_{\notin H}, (-1,1,0) \rangle$



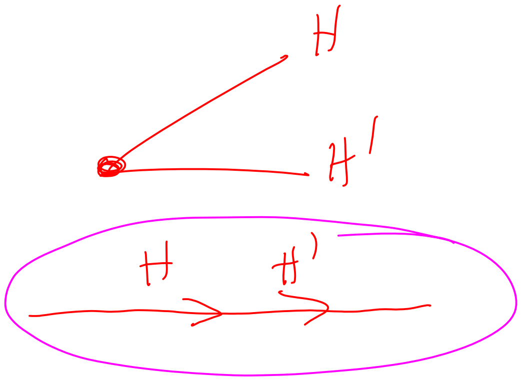
$$H' = \left\{ \begin{vmatrix} x & y & z \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} = 0 \right\} = \{ -y + z + 2z - x = 0 \}$$

$$\boxed{x + y - 3z = 0}$$

$$H \cap H' = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{matrix} x+y=0 \\ x+y-3z=0 \end{matrix} \right\}$$ recta

$$H \cap H' = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & -3 \end{vmatrix} \right\rangle = \langle (1,-1,0) \rangle$$ base

Ej] $H = \langle (1,2,1) \rangle$, $H' = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} 2x - y = 0 \\ y - 2z = 0 \end{array} \right\}$



$H = H' \Rightarrow H \cap H' = H = H'$

$$H = \left\{ \begin{array}{l} 2x - y = 0 \\ y - 2z = 0 \end{array} \right\} \qquad H \cap H' = \left\{ \begin{array}{l} 2x - y = 0 \\ y - 2z = 0 \\ \cancel{2x - y = 0} \\ \cancel{y - 2z = 0} \end{array} \right\}$$

$$\text{rango} \begin{pmatrix} x & y & z \\ 1 & 2 & 1 \end{pmatrix} = 1 \Rightarrow \begin{vmatrix} x & y \\ 1 & 2 \end{vmatrix} = 0, \quad \begin{vmatrix} y & z \\ 2 & 1 \end{vmatrix} = 0$$

Examen enero 2007 | Dados los subespacios vectoriales de $\mathbb{R}^3$:

plano $H_1 = \langle (1,1,-1), (3,0,-1) \rangle$

recta $H_2 = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} -x+y-z=0 \\ 2x-y=0 \end{array} \right\} = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -1 \\ 2 & -1 & 0 \end{vmatrix} \right\rangle = \langle (1,2,1) \rangle$

recta $H_3 = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} 3x+y=0 \\ 2y-z=0 \end{array} \right\} = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 0 & 2 & -1 \end{vmatrix} \right\rangle = \langle (1,-3,-6) \rangle$

a) Calcula una base de $(H_1^\perp + H_2) \cap H_3^\perp$

recta

$H_1^\perp = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} \right\rangle = \langle (-1,-2,-3) \rangle = \langle (1,2,3) \rangle$

plano $H_1^\perp + H_2 = \langle (1,2,3), (1,2,1) \rangle = \left\{ \underbrace{\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} = 0}_{\boxed{-2x+y=0}} \right\} =$

plano $H_3^{\perp} = \left\langle (3,1,0),(0,2,-1) \right\rangle = \left\{ \begin{vmatrix} x & y & z \\ 3 & 1 & 0 \\ 0 & 2 & -1 \end{vmatrix} = 0 \right\} = \{ x-3y-6z=0 \}$

$$\underbrace{\underbrace{\left(H_1^{\perp}+H_2\right)}_{\text{plano}} \cap \underbrace{H_3^{\perp}}_{\text{plano}}}_{\text{recta}} = \left\{ (x,y,z)\in\mathbb{R}^3 : \begin{array}{l} -2x+y=0 \\ x-3y-6z=0 \end{array} \right\} = \left\langle \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 1 & 0 \\ 1 & -3 & -6 \end{vmatrix} \right\rangle$$

$$= \boxed{\left\langle (6,12,5) \right\rangle}$$ base de $(H_1^{\perp}+H_2)\cap H_3^{\perp}$

b) Ecuaciones implícitas de $\underbrace{H_1^{\perp}}_{\text{recta}} + \underbrace{(H_2+H_3)^{\perp}}_{\text{recta}}$

$H_1 = \left\langle (1,1,-1),(3,0,-1) \right\rangle$

$H_2 = \left\{ (x,y,z)\in\mathbb{R}^3 : \begin{array}{l} -x+y-z=0 \\ 2x-y=0 \end{array} \right\}$

$H_3 = \left\{ (x,y,z)\in\mathbb{R}^3 : \begin{array}{l} 3x+y=0 \\ 2y-z=0 \end{array} \right\}$

$H_1^{\perp} = \left\langle (1,2,3) \right\rangle$

$H_2 = \left\langle (1,2,1) \right\rangle$, $H_3 = \left\langle (1,-3,-6) \right\rangle$

$$H_2 + H_3 = \langle (1,2,1), (1,-3,-6) \rangle$$

$$(H_2 + H_3)^{\perp} = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 1 & -3 & -6 \end{vmatrix} \right\rangle = \langle (-9,7,-5) \rangle$$

$$H_1^{\perp} + (H_2 + H_3)^{\perp} = \langle (1,2,3), (-9,7,-5) \rangle = \left\{ \underbrace{\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ -9 & 7 & -5 \end{vmatrix} = 0}_{\boxed{-31x - 22y + 25z = 0}} \right\}$$

plano

Ecuaciones implícitas

Examen Septiembre 2008

plano $H_1 = \langle (1,1,-1), (0,2,-1) \rangle$

plano $H_2 = \{ (x,y,z) \in \mathbb{R}^3 : x - 2y - z = 0 \}$

recta $H_3 = \langle (1,3,-2) \rangle$

recta $H_4 = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{array}{l} y - z = 0 \\ 2x - 3y - 3z = 0 \end{array} \right\} = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 2 & -3 & -3 \end{vmatrix} \right\rangle = \langle (-6,-2,-2) \rangle = \langle (3,1,1) \rangle$

a) Ecuaciones implícitas de $H_1^\perp + (H_3 + H_4)^\perp$

$H_1^\perp = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} \right\rangle = \langle (+1, 1, 2) \rangle$ recta

$(H_3 + H_4)^\perp = \left(\langle (1,3,-2), (3,1,1) \rangle \right)^\perp = \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 3 & 1 & 1 \end{vmatrix} \right\rangle = \langle (5,-7,-8) \rangle$ recta

$$\underbrace{H_1^\perp + (H_2+H_4)^\perp}_{\text{plano}} = \langle (1,1,2), (5,-7,-8) \rangle = \left\{ \begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 5 & -7 & -8 \end{vmatrix} = 0 \right\}$$

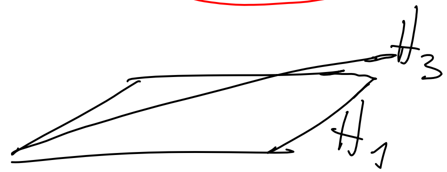
$$\boxed{x + 3y - 2z = 0}$$

b) Una base de $(H_1 \cap H_3)^\perp \cap (H_2 + H_4)$

$$\left[\begin{array}{ll} (H_1 \cap H_3)^\perp \overset{?}{=} H_1^\perp + H_3^\perp \quad (\text{Ejercicio}) & \text{"Leyes de Morgan"} \\ (H_1 + H_3)^\perp \overset{?}{=} H_1^\perp \cap H_3^\perp \quad (\text{Ejercicio}) & \text{"} \end{array} \right]$$

$H_1 \cap H_3 = ?$ ¿$H_3 \subset H_1$? rango $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 3 & -2 \end{pmatrix} = 2$ porque

$$\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix} = 0 \Longrightarrow H_3 \subset H_1$$



$(H_1 \cap H_3 = H_3)^\perp$, $\Longrightarrow$ $H_3^\perp = \{ (x,y,z) \in \mathbb{R}^3 : 1 \cdot x + 3y - 2z = 0 \}$

$(3,1,1)$ verifica las ecuaciones de $H_2 \Rightarrow H_4 \subset H_2 \Rightarrow$

$H_2 + H_4 = H_2$

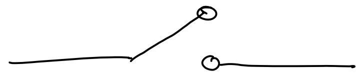
$$\underbrace{(H_1 \cap H_3)^{\perp}}_{H_3^{\perp}} \cap \underbrace{(H_2 + H_4)}_{H_2} = \left\{ \begin{array}{l} x + 3y - 2z = 0 \\ x - 2y - z = 0 \end{array} \right\} =$$

$$= \left\langle \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -2 & -1 \end{vmatrix} \right\rangle = \boxed{\langle (7, 1, 5) \rangle} \text{ Base}$$

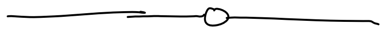
ANALOGÍAS CON LA LÓGICA BOOLEANA

$(0,0,0) = 0$ $\qquad$ $\mathbb{R}^3 = 1$

INTERRUPTOR

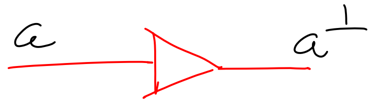


0 ABIERTO

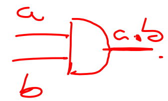


1 CERRADO

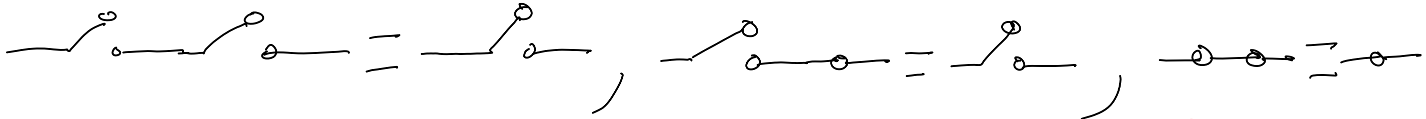
$\boxed{NOT = \perp}$ $\qquad 0^\perp = 1 \qquad 1^\perp = 0$



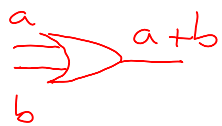
$\boxed{AND = \cap = \cdot}$ $\qquad 0 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 1 \cdot 1 = 1$



Interruptores en serie:



$\boxed{OR = +}$ $\qquad 0 + 0 = 0, \quad 0 + 1 = 1, \quad 1 + 1 = 1$



Interruptores en paralelo: