Taylor-seno

viernes, 17 de octubre de 2008
11:58

Septiembre 2008

a) Estima el valor de $\operatorname{sen}\left(\frac{1}{7}\right)$ utilizando un polinomio de Taylor de grado 4 de la función $f(x) = \operatorname{sen}(x)$ en el punto $x_0 = 0$

b) Usando el resto de Lagrange, calcula qué grado n debe de tener el polinomio de Taylor para que el error cometido sea $E < 10^{-13}$.

a) $$f(x) = \underbrace{f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{(IV)}(x_0)}{4!}(x-x_0)^4}_{P_4(x)} + \underbrace{\frac{f^{(V)}(\theta)}{5!}(x-x_0)^5}_{R_4(x,\theta)}$$

$$f(x)\Big|_{x_0=0} = \operatorname{sen}(x)\Big|_{x=0} = \operatorname{sen}(0) = 0 \qquad f''(x)\Big|_{x=0} = -\operatorname{sen}(x)\Big|_{x=0} = 0$$

$$f'(x)\Big|_{x=0} = \cos(x)\Big|_{x=0} = \cos(0) = 1$$

$$f'''(x)\Big|_{x=0} = -\cos(x)\Big|_{x=0} = -1$$

$$f^{(IV)}(x)\Big|_{x=0} = \operatorname{sen}(x)\Big|_{x=0} = 0$$

$$f^{(V)}(x)\Big|_{x=\theta} = \cos(x)\Big|_{x=\theta} = \cos(\theta)$$

$$P_4(x) = 0 + \frac{1}{1!}x + \frac{-1}{3!}x^3 = x - \frac{x^3}{6}$$

$$\operatorname{sen}\left(\frac{1}{7}\right) \approx \boxed{P_4\left(\frac{1}{7}\right)} = \frac{1}{7} - \frac{(1/7)^3}{6} = \boxed{\frac{293}{2058}} \approx 0'14237123\ldots$$

$$\operatorname{sen}\left(\frac{1}{7}\right) \leq \frac{1}{7}$$

b) $$R_4(x,\theta) = \frac{f^{(V)}(\theta)}{5!}(x-x_0)^5 \qquad 0 = x_0 < \theta < x = \frac{1}{7}$$

$$\left|R_4\left(\frac{1}{7},\theta\right)\right| = \left|\frac{\cos(\theta)}{5!}\left(\frac{1}{7}-0\right)^5\right| < \frac{1}{5!}\left(\frac{1}{7}\right)^5 \approx 4'96 \cdot 10^{-7}$$

$$> 10^{-13}$$

$$\cos(\theta) \begin{cases} \cos(0) = 1 \\ \cos\left(\frac{1}{7}\right) < 1 \end{cases}$$

$$R_5(x,\theta)\Big|_{x=\frac{1}{7}} = \left|\frac{f^{(VI)}(\theta)}{6!}(x-x_0)^6\right| = \left|\frac{-\operatorname{sen}\theta}{6!}(x-x_0)^6\right|_{x=\frac{1}{7}} = \frac{-\operatorname{sen}\theta}{6!}\left(\frac{1}{7}\right)^6$$

$$\Big|_{x=\frac{1}{7}} \quad \frac{\;\cdots\;}{6!} \quad \Big|_{x=\frac{1}{7}} \quad \frac{\;\cdots\;}{6!} \quad \Big|_{x=\frac{1}{7}} \quad \frac{\;\cdots\;}{6!}\left(\frac{1}{7}\right)$$

$$0<\theta<\frac{1}{7} \qquad \operatorname{sen}(\theta) \begin{cases} \to \operatorname{sen}(0) = 0 \\ \to \operatorname{sen}\left(\frac{1}{7}\right) > 0 \end{cases} \qquad \operatorname{sen}\theta < \operatorname{sen}\left(\frac{1}{7}\right)$$

$$\left|R_5\left(\frac{1}{7},\theta\right)\right| \le \frac{\operatorname{sen}\left(\frac{1}{7}\right)}{6!}\left(\frac{1}{7}\right)^6 < \frac{1/7}{6!}\left(\frac{1}{7}\right)^6 \simeq 1'68\cdot 10^{-9} > 10^{-13}$$

$$\operatorname{sen}\left(\frac{1}{7}\right) < \frac{1}{7} < 1$$

$$R_6(x,0) = \frac{f^{(VII)}(\theta)}{7!}(x-x_0)^7 = -\frac{\cos\theta}{7!}(x-x_0)^7\Big|_{x=\frac{1}{7}} = \frac{-\cos\theta}{7!}\left(\frac{1}{7}\right)^7$$

$$0<\theta<\frac{1}{7} \qquad \cos\theta < \cos(0) = 1$$

$$\left|R_6\left(\frac{1}{7},\theta\right)\right| < \frac{1}{7!}\left(\frac{1}{7}\right)^7 \approx 2'4\cdot 10^{-10} > 10^{-13}$$

$$R_7\left(\frac{1}{7},\theta\right) = \frac{\operatorname{sen}\theta}{8!}\left(\frac{1}{7}\right)^8 < \frac{1/7}{8!}\left(\frac{1}{7}\right)^8 \approx 6.1\cdot 10^{-13} > 10^{-13}$$

$$R_8\left(\frac{1}{7},\theta\right) = \frac{\cos\theta}{9!}\left(\frac{1}{7}\right)^9 < \frac{1}{9!}\left(\frac{1}{7}\right)^9 \approx 6.8\cdot 10^{-14} < 10^{-13}$$

$$R_8\left(\frac{1}{7}, \theta\right) = \frac{\cos\theta}{9!}\left(\frac{1}{7}\right)^9 < \frac{1}{9!}\left(\frac{1}{7}\right)^9 \approx \underline{\underline{6.8 \cdot 10^{-14} < 10^{-13}}}$$

$\boxed{n \geq 8}$ Usando el resto de Lagrange

Taylor raiz cúbica

viernes, 17 de octubre de 2008
10:07

Examen de Septiembre 2007

a) Estima el valor de $\sqrt[3]{3/2}$ utilizando un polinomio de Taylor de grado 3 de la función $f(x) = \sqrt[3]{x}$ en el punto $x_0 = 1$.

b) Usando el resto de Lagrange, calcula qué grado n debe tener el polinomio de Taylor para que el error cometido sea menor $E < 10^{-4}$.

a) $$f(x) = \underbrace{f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3}_{P_3(x)} + \underbrace{\frac{f^{(IV)}(\theta)}{4!}(x-x_0)^4}_{R_3(x,\theta)}$$

$f(x)| = \sqrt[3]{x}| = x^{1/3}| = 1 \qquad f'(x)| = 1\, x^{-\frac{2}{3}}| = 1$

$$f(x)\Big|_{x=x_0} = \sqrt[3]{x}\Big|_{x=1} = x^{1/3}\Big|_{x=1} = 1, \quad f'(x)\Big|_{x=1} = \frac{1}{3}x^{-\frac{2}{3}}\Big|_{x=1} = \frac{1}{3}$$

$$f''(x)\Big|_{x=1} = \frac{1}{3}\left(-\frac{2}{3}\right)x^{-5/3}\Big|_{x=1} = \frac{-2}{9}, \quad f'''(x)\Big|_{x=1} = \left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)x^{-8/3}\Big|_{x=1} = \frac{10}{27}$$

$$f^{(IV)}(\ ) = \frac{10}{27}\cdot\left(-\frac{8}{3}\right)x^{-11/3} = \frac{-80}{27\cdot 3}x^{-11/3}$$

$$P_3(x) = 1 + \frac{1}{3}(x-1) - \frac{2/9}{2}(x-1)^2 + \frac{10/27}{6}(x-1)^3$$

$$\sqrt[3]{3/2} \approx \boxed{P_3\left(\frac{3}{2}\right)} = 1 + \frac{1}{3}\underbrace{\left(\frac{3}{2}-1\right)}_{1/2} - \frac{1}{9}\underbrace{\left(\frac{3}{2}-1\right)}_{1/2}^2 + \frac{10/27}{6}\left(\frac{3}{2}-1\right)^3$$

$$= \boxed{\frac{743}{648}}$$

b) $$R_3(x,\theta) = \frac{f^{(IV)}(\theta)}{4!}(x-x_0)^4 \qquad \underline{\underline{1}} = x_0 < \theta < x = \underline{\underline{\frac{3}{2}}}$$

$$f^{(IV)}(\theta) = \frac{-80}{27\cdot 3}\cancel{\theta}^{-11/3} = \frac{-80}{27\cdot 3}\frac{1}{\theta^3\,\theta^{2/3}}$$

$$f^{(IV)}(\theta) = \frac{-80}{27\cdot 3}\theta^{-11/3} = \frac{-80}{27\cdot 3}\frac{1}{\theta^3\,\theta^{2/3}}$$

$$\theta^{-11/3} = \boxed{\frac{1}{\theta^{11/3}} < 1}$$

$$\theta = 1 \rightarrow 1$$

$$\theta = \frac{3}{2} \rightarrow \frac{1}{\left(\frac{3}{2}\right)^{11/3}} < 1$$

$$\left|R_3\left(\tfrac{3}{2},\theta\right)\right| = \left|\frac{\frac{-80}{27\cdot 3}\overbrace{\theta^{-11/3}}^{<1}}{4!}\underbrace{\left(\frac{3}{2}-1\right)}_{1/2}^{4}\right| < \frac{\frac{80}{27\cdot 3}\cdot 1}{4!}\left(\frac{1}{2}\right)^4 = 0'0026 > 10^{-4}$$

$$\sqrt[3]{\frac{3}{2}} = \frac{743}{648} \pm 0'0026$$

$$R_4(x,\theta)\Big|_{x=\frac{3}{2},\,x_0=1} = \frac{f^{(V)}(\theta)}{5!}(x-x_0)^5\Big|_{x=\frac{3}{2},\,x_0=1} = \frac{\frac{-80}{27\cdot 3}\left(\frac{-11}{3}\right)\theta^{-14/3}}{5!}(x-x_0)^5\Big|_{x=\frac{3}{2},\,x_0=1} =$$

$$= \frac{80\cdot 11}{5!\,27\cdot 9}\theta^{-14/3}\underbrace{\left(\frac{3}{2}-1\right)}_{1/2}^{5} \qquad 1 < \theta < \frac{3}{2}$$

$$\theta^{-14/3} = \frac{1}{\ldots} \quad \theta = 1 \rightarrow 1 \quad < 1$$

$$\theta^{-14/3} = \frac{1}{\theta^{14/3}} \quad \begin{cases} \theta = 1 \to 1 \\ \theta = \frac{3}{2} \to \dfrac{1}{\left(\frac{3}{2}\right)^{14/3}} < 1 \end{cases} < 1$$

$$\left| R_4\left(\tfrac{3}{2}, \theta\right) \right| < \frac{80 \cdot 11}{5! \cdot 27 \cdot 9} \cdot 1 \left(\frac{1}{2}\right)^5 \approx 0'00094 > 10^{-4}$$

$$\left| R_5\left(\tfrac{3}{2}, \theta\right) \right| = \left| \frac{f^{(VI)}(\theta)}{6!} \left(\tfrac{3}{2} - 1\right)^6 \right| = \left| \frac{\frac{-80}{27 \cdot 3} \cdot \frac{11}{3} \cdot \frac{14}{3} \theta^{-17/3}}{6!} \left(\frac{1}{2}\right)^6 \right| < \quad \theta^{-17/3} < 1$$

$$< \frac{\frac{80 \cdot 11 \cdot 14}{27 \cdot 3 \cdot 3 \cdot 3}}{6!} \left(\frac{1}{2}\right)^6 = \frac{0'00094}{6} \cdot \frac{14}{3} \cdot \frac{1}{2} \approx 0'00037 > 10^{-4}$$

$$\left| R_6\left(\tfrac{3}{2}, \theta\right) \right| = \left| \frac{f^{(VII)}(\theta)}{7!} \left(\frac{1}{2}\right)^7 \right| < \frac{0'00037}{7} \cdot \frac{17}{3} \cdot \left(\frac{1}{2}\right) \approx 0'00015 > 10^{-4}$$

$$\left| R_7\left(\tfrac{3}{2}, \theta\right) \right| \; \cdots\cdots \; < \frac{0'00015}{8} \cdot \frac{20}{3} \left(\frac{1}{2}\right) \approx 0'00006 < 10^{-4}$$

$$\boxed{n \geq 7}$$

O.K.

$\boxed{n = +}$

O.K.

Taylor logaritmo

miércoles, 22 de octubre de 2008
10:05

Examen de Junio 2008

a) Estima el valor de $\ln\left(\frac{9}{8}\right)$ mediante un polinomio de Taylor de grado 4 de la función $f(x) = \ln(x)$ en el pto $x_0 = 1$

b) Usando el resto de Lagrange, calcula qué grado n debe de tener el polinomio de Taylor para que el error cometido sea $E < 5 \cdot 10^{-7}$

$$f(x) = \underbrace{f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{(IV)}(x_0)}{4!}(x-x_0)^4}_{P_4(x)} + \underbrace{\frac{f^{(V)}(\theta)}{5!}(x-x_0)^5}_{R_4(x,\theta)}$$

a) $f(x_0) = \ln(x_0) = \ln(1) = 0$

$f'(x)\Big|_{x=1} = \frac{1}{x}\Big|_{x=1} = 1$ $\quad (x^{-1})$

$f'''(x)\Big|_{x=1} = 2x^{-3}\Big|_{x=1} = 2$

$f^{(IV)}(x)\Big|_{x=1} = -6x^{-4}\Big|_{x=1} = -6$

$$\Big|_{x=1} = 2$$

$$f'(x)\Big|_{x=1} = \left(\frac{1}{x}\right)\Big|_{x=1} = 1 \qquad f^{(iv)}(x)\Big|_{x=1} = -6x^{-4}\Big|_{x=1} = -6$$

$$f''(x)\Big|_{x=1} = -1\,x^{-2}\Big|_{x=1} = -1 \qquad f^{(v)}(x)\Big|_{x=\theta} = 24x^{-5}\Big|_{x=\theta} = 24\,\theta^{-5}$$

$$P_4(x) = 0 + \frac{1}{1!}(x-1) + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4$$

$$\ln\left(\frac{9}{8}\right) \approx P_4\left(\frac{9}{8}\right) = \underbrace{\left(\frac{9}{8}-1\right)}_{1/8} - \frac{1}{2}\left(\frac{9}{8}-1\right)^2 + \frac{1}{3}\left(\frac{9}{8}-1\right)^3 - \frac{1}{4}\left(\frac{9}{8}-1\right)^4$$

$$\boxed{P_4\left(\frac{9}{8}\right)} = \frac{1}{8} - \frac{1}{2}\left(\frac{1}{8}\right)^2 + \frac{1}{3}\left(\frac{1}{8}\right)^3 - \frac{1}{4}\left(\frac{1}{8}\right)^4 = \boxed{\frac{5789}{49152}}$$

b) $$R_4(x,\theta)\Big|_{\substack{x=9/8\\ x_0=1}} = \frac{f^{(v)}(\theta)}{5!}(x-x_0)^5 = \frac{24\,\theta^{-5}}{5!}\left(\frac{1}{8}\right)^5 < \frac{24\cdot 1}{5!}\left(\frac{1}{8}\right)^5 \approx 6'1\cdot 10^{-6}$$

$$> 5\cdot 10^{-7}$$

$$x_0 = 1 < \theta < x = 9/8 \qquad \theta^{-5} = \frac{1}{\theta^5} \xrightarrow{\theta=1} 1 \qquad \frac{1}{(9/8)^5} < 1 \qquad 1 < \theta < \frac{9}{8}$$

$$\left|R_5\left(\frac{9}{8},\theta\right)\right| = \left|\frac{f^{(vi)}(\theta)}{6!}\left(\frac{1}{8}\right)^6\right| = \left|\frac{-120\cdot\theta^{-6}}{6!}\left(\frac{1}{8}\right)^6\right| < \left|\frac{-120\cdot 1}{6!}\left(\frac{1}{8}\right)^6\right| =$$

$$= \frac{6'1\cdot 10^{-6}\cdot 5}{6}\left(\frac{1}{8}\right) \approx 6'4\cdot 10^{-7} > 5\cdot 10^{-7}$$

$$\left|R_6\left(\frac{9}{8},\theta\right)\right| = \left|\frac{f^{(vii)}(\theta)}{[illegible]}\left(\frac{1}{8}\right)^7\right| = \left|\frac{6\cdot 120\,\theta^{-7}}{[illegible]}\left(\frac{1}{8}\right)^7\right| < \left|\frac{6\cdot 120}{[illegible]}\left(\frac{1}{8}\right)^7\right| =$$

$$\theta^{-7} < 1$$

$$\left|R_6\left(\frac{9}{8},\theta\right)\right| = \left|\frac{f^{(VII)}(\theta)}{7!}\left(\frac{1}{8}\right)^7\right| = \left|\frac{6\cdot 120\,\theta^{-7}}{7!}\left(\frac{1}{8}\right)^7\right| < \left|\frac{6\cdot 120}{7!}\left(\frac{1}{8}\right)^7\right| =$$

$$6'4\cdot 10^{-7}\cdot\frac{6}{7}\left(\frac{1}{8}\right) \approx 6'8\cdot 10^{-8} < 5\cdot 10^{-7}$$

$$\boxed{n \geq 6}$$

7a)

$\ln(2) \qquad x_0 = 1 \qquad n : \quad E < 10^{-6}$

$$\ln''''(x) = (x^{-1})''' = (-1x^{-2})'' = ((-1)(-2)x^{-3})' = (-1)(-2)(-3)x^{-4}$$

$$\boxed{\ln^{(n)}(x) = (-1)\cdot(-2)\cdot(-3)\cdots\cdots(-(n-1))\,x^{-n} = (n-1)!\,(-1)^n x^{-n}}$$

$$\boxed{R_n(x,\theta) = \frac{f^{(n+1)}(\theta)}{(n+1)!}(x-x_0)^{n+1} = \frac{n!\,(-1)^{n+1}\,\theta^{-(n+1)}}{(n+1)!}(x-x_0)^{n+1}}$$

$\theta^{-(n+1)} < 1$

$$|R_n(x,\theta)| < \frac{n!}{(n+1)!}(x-x_0)^{n+1} = \frac{1}{n+1} \leq 10^{-6}$$

$x \to 2, \quad x_0 \to 1$

$$10^6\left(\frac{1}{\ldots}\right) < n+1 \quad \Rightarrow \quad n \geq 10^6 - 1 = 999.999$$

$$10^{6}\left(\frac{1}{10^{-6}}\right) < n+1 \implies n \geq 10^{6}-1 = 999.999$$

Taylor raiz cuadrada

miércoles, 22 de octubre de 2008
10:56

$\sqrt{2}$, $\sqrt{0.5}$ sabiendo $\sqrt{1} = 1$ $\sqrt{5}$, $\sqrt{3}$; $\sqrt{4} = 2$

a) Estima el valor de $\sqrt{2}$ usando un polinomio de Taylor de grado 3 de la función $f(x) = \sqrt{x}$ en el pto. $x_0 = 1$

b) Usando el resto de Lagrange, da un cota del error cometido.

$$f(x) = \underbrace{f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3}_{P_3(x)} + \underbrace{\frac{f^{(IV)}(\theta)}{4!}(x-x_0)^4}_{R_3(x,\theta)}$$

a) $f(x)\Big|_{x=x_0} = \sqrt{x}\Big|_{x=1} = \sqrt{1} = 1$ $\quad \sqrt{x} = x^{1/2}$

$f''(x)\Big|_{x=1} = \frac{1}{2}\left(-\frac{1}{2}\right)x^{-3/2}\Big|_{x=1} = -\frac{1}{4}$

$f'(x)\Big| = \frac{1}{2}x^{\frac{1}{2}-1}\Big| = \frac{1}{2}x^{-1/2}\Big| = \frac{1}{2}$

$f'''(x)\Big| = \left(\frac{1}{4}\right)\left(-\frac{3}{2}\right)x^{-5/2}\Big| = \frac{3}{8}$

$$f'(x)\Big|_{x=1} = \frac{1}{2}x^{\frac{1}{2}-1}\Big|_{x=1} = \frac{1}{2}x^{-1/2}\Big|_{x=1} = \frac{1}{2} \quad\Big|\quad f'''(x)\Big|_{x=1} = \left(-\frac{1}{4}\right)\left(-\frac{3}{2}\right)x^{-5/2}\Big|_{x=1} = \frac{3}{8}$$

$$P_3(x) = 1 + \frac{1/2}{1!}(x-1) + \frac{-1/4}{2!}(x-1)^2 + \frac{3/8}{3!}(x-1)^3$$

$$\sqrt{2} \approx P_3(2) = 1 + \frac{1}{2}(\underbrace{2-1}_{1}) - \frac{1}{8}(2-1) + \frac{1}{16}(2-1)^3 =$$

$$= 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16} = \boxed{\frac{23}{16}} \approx 1'4375$$

$$\begin{array}{r} 1'4 \\ 1'4 \\ \hline 1'96 \end{array}$$

b) $$R_3(x,\theta)\Big|_{\substack{x=2\\x_0=1}} = \frac{f^{(N)}(\theta)}{4!}(\underbrace{x-x_0}_{2-1})^4 = \frac{\frac{3}{8}\left(-\frac{5}{2}\right)\theta^{-7/2}}{4!}(2-1)^4 = \frac{-15}{16\cdot 24}\theta^{-7/2}$$

$$\underline{\underline{1}} = x_0 < \theta < x = \underline{\underline{2}}$$

$$\theta^{-7/2} = \frac{1}{\theta^{7/2}} \quad \theta=1 \to 1 < 1, \quad \theta=2 \to \frac{1}{2^{7/2}}$$

$$\left|R_3(2,\theta)\right| < \frac{15}{16\cdot 24}\cdot 1 \approx 0'04$$

$$\sqrt{2} = \frac{23}{\ } + 0'04$$

$$\left|R_3(x,\theta)\right| < \frac{\quad}{16}\cdot 1 \approx 0'0\ldots$$

$$\boxed{\sqrt{2} = \frac{23}{16} \pm 0'04}$$

a) Estima $\sqrt{0'5}$ mediante un polinomio de Taylor de grado 3 de la función $f(x)=\sqrt{x}$ en el pto. $x_0=1$

b) Calcula el error usando Lagrange.

a) $$P_3(x) = 1 + \frac{1/2}{1!}(x-1) + \frac{-1/4}{2!}(x-1)^2 + \frac{3/8}{3!}(x-1)^3$$

$$\sqrt{0'5} \approx \boxed{P_3(0'5)} = 1 + \frac{1}{2}(0'5-1) - \frac{1}{8}(0'5-1)^2 + \frac{1}{16}(0'5-1)^3 =$$

$$= \boxed{\frac{91}{128}} \approx 0'710938$$

b) $$R_3(x,\theta)\Big|_{x=0'5} = \frac{f^{(IV)}(\theta)}{4!}(x-x_0)^4\Big|_{x=0'5} = \frac{-15}{16\cdot 24}\,\theta^{-7/2}\,(0'5-1)^4$$

$\theta^{-7/2} < 8\cdot 1'5$

$$\underset{x}{0'5} < \theta < \underset{x_0}{1}$$

$$\theta^{-7/2} = \frac{1}{\theta^{7/2}} \quad \theta=1 \to 1, \quad \theta=0'5 \to \frac{1}{\left(\frac{1}{2}\right)^{7/2}} = 2^{7/2} = \underset{8}{2^3}\,2^{1/2}$$

$\boxed{\sqrt{2} < 1'5}$ $2^{1/2} < 1'5$

$-7/2$ $\quad 3 \quad 1$

$\hat{}$ $\quad x_0 \quad \theta^{7/2} \quad 1$

$\theta = 0'5 \quad \left(\frac{1}{2}\right)^{7/2} = \ldots$

$\boxed{\sqrt{2} < 1'5} \quad \Rightarrow < 1'5$

$\theta^{-7/2} < 2^3 \cdot 1'5$

$1'42$

$$|R_3(0'5,\theta)| < \left| \frac{-15}{16 \cdot 24} \, 8 \cdot 1'5 \left(\frac{-1}{2}\right)^4 \right| \approx 0'03$$

$$\begin{array}{r} 1'42 \\ 1'42 \\ \hline 2'0164 \end{array}$$

$\cancel{\sqrt{2} \simeq 1'41421}$

$$\boxed{\sqrt{0'5} = \frac{91}{128} \pm 0'03}$$

$\boxed{\sqrt{2} < 1'42}$

Otra cota