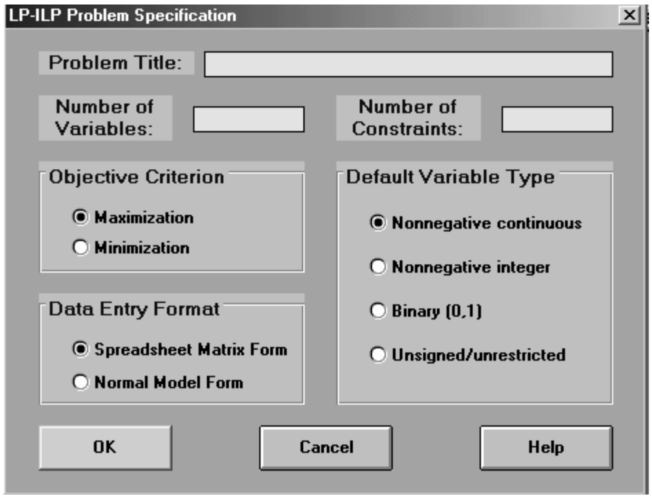
USING WINQSB TO SOLVE LINEAR PROGRAMMING PROBLEMS

1. CREATING A NEW LINEAR OR INTEGER PROGRAMMING PROBLEM

The ***New Problem*** option generates a template for inserting the characteristics of our problem:



Each of the boxes of this window is described below:

- ***Problem Title***: Enter the title used to identify the problem.
- ***Number of Variables***: Enter the number of system variables in the original model.
- ***Number of Constraints***: Enter the number of constraints of the model (should not include the nonnegative constraint).
- ***Objective*** (***Objective Criterion***): There are two classifications for linear and integer programming problems: ***Maximization*** and ***Minimization problems.***
- ***Data Entry Format:*** This option allows choosing between two different types of templates for entering model data. The first alternative is similar to a spreadsheet whereas the second is a template specially designed for this purpose.
- ***Type of Variable*** (***Default Variable Type***): The model characteristics are indicated in this section:
  - ***Nonnegative continuous***: Indicates that the model consists of nonnegative continuous variables (equal or greater than zero).
  - ***Nonnegative Integer***: Nonnegative integer variables.
  - ***Binary:*** Variables which only have a value of 0 or 1.
  - (***Unsigned/unrestricted***): Unrestricted variables.

2. SAMPLE PROBLEM

By means of a sample problem, we will show how data are entered to create a new linear programming problem.

EXAMPLE

The firm **ABC S.A.** wants to know the number of A, B, and C products that it should produce in order to maximise profit taking into account that each unit sold generates a utility of 150€, 210€ and 130€ per unit, respectively. Each product goes through 3 different working tables, restricting the number of units produced because of the time available at each of the tables. The following chart shows the time required for each product unit at each table and the total time available during the week (time expressed in minutes):

	Required time Table 1	**Required time Table 2**	**Required time Table 3**
Product 1	10	12	8
Product 2	15	17	9
Product 3	7	7	8
Total available time by table	3300	3500	2900

It is assumed that each unit produced is sold automatically. Determine the combination of products required to maximise the firm's profit. After analysing the example, the reader will create a mathematical model.

Decision variables:

X1=number of product 1 units to produce

X2=number of product 2 units to produce

X3=number of product 3 units to produce

Objective Function:

Max. Z = 150*X1 + 210*X2 + 130*X3

Restrictions:

10*X1 + 15*X2 + 7*X3 ≤ 3300 (Minutes)

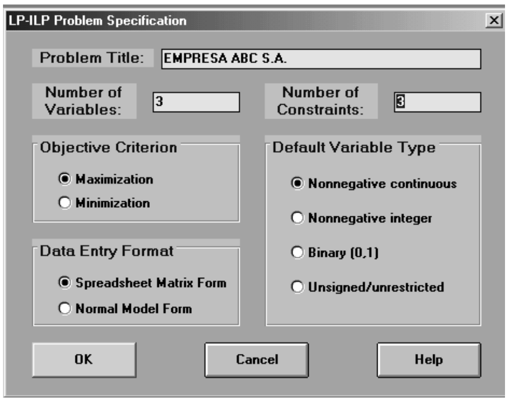
12*X1 + 17*X2 + 7*X3 ≤ 3500 (Minutes)

8*X1 + 9*X2 + 8*X3 ≤ 2900 (Minutes)

X1, X2, X3 ≥ 0

First, we must establish that this is a Maximization product with three constraints and three variables (which we will use as continuous nonnegative variables).

Once this is clear, feed the program from the **New Problem** window.



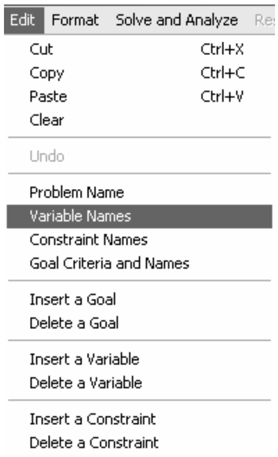
After all the fields have been filled in, click on the ***OK*** button to generate new options within the program.

3. INSERTING THE MODEL

If the **Spreadsheet Matrix Form** is chosen, a new window will open in the working area to insert the mathematical model.

Variable -->	X1	X2	X3	Direction	R. H. S.
Maximize					
C1				<=	
C2				<=	
C3				<=	
LowerBound	0	0	0		
UpperBound	M	M	M		
VariableType	Continuous	Continuous	Continuous		

The first row (***Variable -->***) is the heading of the variables which the system defines automatically as X1, X2 and X3 (the three sample variables), followed by the relationship operator (**Direction**) and the constraints solution or ***Right Hand Side*** -***R. H. S***. The name of the variables can be changed in the ***Variables Names*** submenu from the **Edit** menu.



The second row (***Maximize***) is for inserting the coefficients of the objective function. Several rows identified with the letter **Ci** will appear followed by a consecutive corresponding to the number of constraints of the model (C1 indicates “Constraint 1”, etc.)

C1				<=	
C2				<=	
C3				<=	

Lastly, three rows appear where the minimum accepted value for each variable (***Lower Bound***), the maximum value (***Upper Bound***) and the type of variable (***Variable Type***) are defined. In the case of the maximum value, ***M*** means that the variable can receive very large values (tending to be infinite).

4. THE SAMPLE MODEL

To enter the model proposed in the example, firstly insert the coefficients of the objective function in the second row:

Variable -->	X1	X2	X3	Direction	R. H. S.
Maximize	150	210	130		

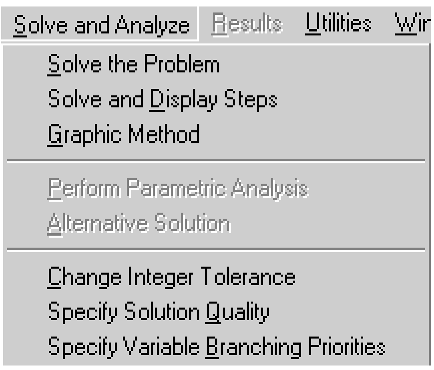
Then enter the coefficients of the constraints C1, C2 and C3:

C1	10	15	7	<=	3300
C2	12	17	7	<=	3500
C3	8	9	8	<=	2900

To change the relationship operators, click twice on top using the left-hand side of the mouse. The other rows remain unchanged.

5. SOLVING THE PROBLEM

After inserting the model in the template, use the tools provided in the **Solve and Analyze** menu. The menu offers the following options:



- ***Solve the Problem***: Solves the problem by using the Simplex Primal method. It shows the final complete solution.
- ***Solve and Display Steps***: Shows each of the steps or interactions performed by Simplex until achieving the optimal solution.
- ***Graphic Method***: Solves the linear programming problem using the graphic method (for problems using two variables).

For the example problem, select the first option in the **Solve and Analyze** menu. A small window will pop up with the message: "**The problem has been solved. Optimal solution is achieved".**

Click on the **Accept** button and the program will automatically generate the optimal solution.

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X1	0	150.0000	0	-14.9315	at bound	-M	164.9315
X2	105.4795	210.0000	22,150.6900	0	basic	182.7500	315.7143
X3	243.8356	130.0000	31,698.6300	0	basic	91.0714	186.6667
Objective	Function	(Max.) =	53,849.3200				
Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
C1	3,289.0410	<=	3,300.0000	10.9589	0	3,289.0410	M
C2	3,500.0000	<=	3,500.0000	0	6.9863	2,537.5000	3,514.0350
C3	2,900.0000	<=	2,900.0000	0	10.1370	1,852.9410	2,957.1430

6. UNDERSTANDING THE RESULTS (COMBINED REPORT)

This matrix displays sufficient information on the solved model. The upper section relates to the analysis of the defined variables (X1, X2 and X3).

Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
X1	0	150.0000	0	-14.9315	at bound	-M	164.9315
X2	105.4795	210.0000	22,150.6900	0	basic	182.7500	315.7143
X3	243.8356	130.0000	31,698.6300	0	basic	91.0714	186.6667
Objective	Function	(Max.) =	53,849.3200				

The ***Solution Value*** column displays the optimal values that were found. In this example, X1 represents 0 units, X2 105.4795 units and X3 is 243.8356 units.

The ***Unit Cost or Profit*** column displays the initial coefficients of each variable in the objective function.

The ***Total Contribution*** column displays the cost or profit (or whatever the objective is) generated by each variable. For example, as the value of the X2 variable is 105.4795 units and the profit per unit is 210€, the total profit will be the result of multiplying both values, arriving at the figure of 22,150.69€. The optimal objective value (53,849.32€) appears just below the last contribution.

The ***Reduced Cost*** column identifies the cost generated by increasing one unit for each at bound variable. The following column, ***Basis Status,*** indicates if a variable is ***basic*** or non basic (indicating that it's non basic as ***at bound***).

The following section of the final matrix (***Constraints Summary***), displays the system's dummy variables (slack or surplus).

	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	3,289.0410	<=	3,300.0000	10.9589	0	3,289.0410	M
2	C2	3,500.0000	<=	3,500.0000	0	6.9863	2,537.5000	3,514.0350
3	C3	2,900.0000	<=	2,900.0000	0	10.1370	1,852.9410	2,957.1430

The ***Left Hand Side*** column displays the value achieved by replacing X1, X2 and X3 values in each restriction (bear in mind that each restriction is identified with its corresponding dummy variable).
The following two columns (***Direction*** and ***Right Hand Side***) display the specifications given to the constraints in terms of the relationship operator (≤) and the original values of the constraints (3,300, 3,500 and 2,900 minutes).

The ***Slack or Surplus*** columns show the values of the dummy variables and the ***Shadow Price*** column relates to the shadow prices: ***how much would you be willing to pay for an additional unit of each resource?***

7. THE FINAL SIMPLEX TABLE

WINQSB makes it possible to display the optimal results by means of the format applied by the Simplex method. To be able to display this format, once the problem has been solved, select the **Final Simplex Tableau** option from the **Results** menu.

		X1	X2	X3	Slack_C1	Slack_C2	Slack_C3		
Basis	C(j)	150.0000	210.0000	130.0000	0	0	0	R. H. S.	Ratio
Slack_C1	0	-0.9041	0.0000	0.0000	1.0000	-0.7808	-0.1918	10.9589	
X2	210.0000	0.5479	1.0000	0.0000	0	0.1096	-0.0959	105.4795	
X3	130.0000	0.3836	0.0000	1.0000	0	-0.1233	0.2329	243.8356	
	C(j)-Z(j)	-14.9315	0	0	0	-6.9863	-10.1370	53,849.3200	