Mathematics for Business I

1^{st} G.A.D.E., Academic Year 2011/12

Control Unit 2 (Option B)

SURNAME(S): .... NAME: ....

1. Let $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear map such that $(1,0,0)$ is an eigenvector with eigenvalue $\lambda = 6$, $f(0,1,0) = (2,6,0)$ and $f(0,0,1) = (4,8,10)$. Let $M(f)$ the matrix associated to f in the canonical basis.

   a) Calculate $M(f)$.

   b) Calculate the eigenvalues of f, its multiplicities, and an eigenvector of each eigenvalue (different from $(1,0,0)$).

   c) Study if f is diagonalizable.

   Solución:

   a) $M(F) = \begin{pmatrix} 6 & 2 & 4 \\ 0 & 6 & 8 \\ 0 & 0 & 10 \end{pmatrix}$.

   b) It has two eigenvalues: 6 (with multiplicity $m_1 = 2$), and 10 (with multiplicity $m_2 = 1$).
   $(-2,-2,-1)$ is an eigenvector of the eigenvalue 10. $(3,0,0)$ is another eigenvector of the eigenvalue 6.

   c) It is not diagonalizable, because $dim\ H(3) = 1 \neq m_1$.

2. The return of an investment, R, depends on three financial parameters x, y, z, through the relationship $R(x,y,z) = x^2+3y^2+(k+1)z^2+2kyz+2xz$, where $k \in \mathbb{R}$.

a) Find a value for the parameter k, such that the investment is always profitable (i.e., the return always positive).

b) Find a value for the parameter k, such that the investment is always profitable or null.

c) Find a value for the parameter k, such that the investment is sometimes profitable and sometimes non profitable.

d) Consider $k = 4$ and study if the investment is profitable or not when $y = x - z$.

e) Consider $k = 4$ and study if the investment is profitable or not when $y = x - z$ and $x = 3z$.

Solución:

a) The minors of the matrix are $A_1 > 0$, $A_2 > 0$, $A_3 = 3k - k^2 = k(3-k)$. $A_3 > 0$ when $0 < k < 3$. So it could be $k = 2$, for example.

b) $A_3 = 0$ when $k = 0$ or $k = 3$.

c) $A_3 < 0$ when $k < 0$ or $k > 3$. So it could be $k = 4$, for example.

d) R is indefinite in that case, so it is sometimes profitable and sometimes not.

e) R is positive definite in that case, so yes, it is profitable.