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EXERCISE 1(a)

#1: $$\frac{x^2 + y^2}{z\cdot(x + y)}$$

#2: $$\mathrm{GRAD}\left(\frac{x^2 + y^2}{z\cdot(x + y)}, [x, y, z]\right)$$

#3: $$\left[\frac{x^2 + 2\cdot x\cdot y - y^2}{z\cdot(x + y)^2}, -\frac{x^2 - 2\cdot x\cdot y - y^2}{z\cdot(x + y)^2}, -\frac{x^2 + y^2}{z^2\cdot(x + y)}\right]$$

Now we replace the values x=2, y=-3/4, z=1:

#4: $$\left[\frac{2^2 + 2\cdot 2\cdot\left(-\frac{3}{4}\right) - \left(-\frac{3}{4}\right)^2}{1\cdot\left(2 + -\frac{3}{4}\right)^2}, -\frac{2^2 - 2\cdot 2\cdot\left(-\frac{3}{4}\right) - \left(-\frac{3}{4}\right)^2}{1\cdot\left(2 + -\frac{3}{4}\right)^2}, -\frac{2^2 + \left(-\frac{3}{4}\right)^2}{1^2\cdot\left(2 + -\frac{3}{4}\right)}\right]$$

#5: $$\left[\frac{7}{25}, -\frac{103}{25}, -\frac{73}{20}\right]$$

EXERCISE 1(b)

#6: $$\mathrm{GRAD}\left(\frac{x\cdot y\cdot z^3\cdot(x^3 - y^2 - z^2)}{x^3 + y^2 + z^2}, [x, y, z]\right)$$

#7: $$\left[\frac{y\cdot z^3\cdot(x^6 + 6\cdot x^3\cdot(y^2 + z^2) - (y^2 + z^2)^2)}{(x^3 + y^2 + z^2)^2},\right.$$

$$\frac{x\cdot z^3\cdot(x^6-4\cdot x^3\cdot y^2-(y^2+z^2)^2)}{(x^3+y^2+z^2)^2},$$

$$\left.\frac{x\cdot y\cdot z^2\cdot(3\cdot x^6-4\cdot x^3\cdot z^2-3\cdot(y^2+z^2)^2)}{(x^3+y^2+z^2)^2}\right]$$

#8: $$\left[\frac{393421}{376996}, -\frac{141104026}{282747}, \frac{109701347}{565494}\right]$$

#9: $$\frac{x\cdot y\cdot z^3\cdot(x^3-y^2-z^2)}{x^3+y^2+z^2}$$

EXERCISE 1(c)

#10: $$\mathrm{GRAD}\left(x\cdot y\cdot e^{x^2}+e^y\cdot z\cdot e^x, [x, y, z]\right)$$

#11: $$\left[y\cdot e^{x^2}\cdot(2\cdot x^2+1)+z\cdot e^{x+y}, x\cdot e^{x^2}+z\cdot e^{x+y}, e^{x+y}\right]$$

#12: $$\left[y\cdot e^{x^2}\cdot(2\cdot x^2+1)+z\cdot e^{x+y}, x\cdot e^{x^2}+z\cdot e^{x+y}, e^{x+y}\right]$$

#13: $$\left[e^{5/4}-\frac{27\cdot e^4}{4}, 2\cdot e^4+e^{5/4}, e^{5/4}\right]$$

EXERCISE 1(e)

#14: $$\mathrm{GRAD}\left(\frac{\mathrm{LN}(r+t)}{r+s+t}, [r, s, t]\right)$$

#15: $$\left[-\frac{(r+t)\cdot\mathrm{LN}(r+t)-r-s-t}{(r+s+t)^2\cdot(r+t)}, -\frac{\mathrm{LN}(r+t)}{(r+s+t)^2}, -\right.$$

$$\left.\frac{(r+t)\cdot\mathrm{LN}(r+t)-r-s-t}{(r+s+t)^2\cdot(r+t)}\right]$$

#16: $$\left[\frac{4}{27} - \frac{16\cdot LN(3)}{81}, - \frac{16\cdot LN(3)}{81}, \frac{4}{27} - \frac{16\cdot LN(3)}{81}\right]$$

EXERCISE 1(d)

#17: $$GRAD((x + y)\cdot e^{y\cdot z}, [x, y, z])$$

#18: $$\left[e^{y\cdot z}, e^{y\cdot z}\cdot(x\cdot z + y\cdot z + 1), y\cdot e^{y\cdot z}\cdot(x + y)\right]$$

#19: $$\left[e^{- 3/4}, \frac{9\cdot e^{- 3/4}}{4}, - \frac{15\cdot e^{- 3/4}}{16}\right]$$

EXERCISE 1(f)

#20: $$GRAD\left(\left[z\cdot t^{3}\cdot LOG(x + y) + x\cdot e^{x^{2}\cdot y} + \sqrt{(u\cdot z)}\right], [x, y, z, t, u]\right)$$

#21: $$\left[\begin{array}{c} e^{x^{2}\cdot y}\cdot(2\cdot x^{2}\cdot y + 1) + \dfrac{t^{3}\cdot z}{x + y} \\ x^{3}\cdot e^{x^{2}\cdot y} + \dfrac{t^{3}\cdot z}{x + y} \\ t^{3}\cdot LN(x + y) + \dfrac{\sqrt{(u\cdot z)}}{2\cdot z} \\ 3\cdot t^{2}\cdot z\cdot LN(x + y) \\ \dfrac{\sqrt{(u\cdot z)}}{2\cdot u} \end{array}\right]$$

EXERCISE 1(g)

#22: $$GRAD\left(\left[y^{4\cdot x} + \frac{e\cdot(4\cdot z)}{\sqrt{(x\cdot y)}} + \frac{SIN(u)}{COS(t)}\right], [x, y, z, t, u]\right)$$

#23:

$$\left[\begin{array}{c} 4\cdot y^{4\cdot x}\cdot LN(y) - \dfrac{2\cdot e\cdot z\cdot\sqrt{(x\cdot y)}}{x^{2}\cdot y} \\ 4\cdot x\cdot y^{4\cdot x - 1} - \dfrac{2\cdot e\cdot z\cdot\sqrt{(x\cdot y)}}{x\cdot y^{2}} \\ \dfrac{4\cdot e}{\sqrt{(x\cdot y)}} \\ \dfrac{SIN(t)\cdot SIN(u)}{COS(t)^{2}} \\ \dfrac{COS(u)}{COS(t)} \end{array}\right]$$

#24:

$$\left[\begin{array}{c} - 4\cdot e \\ 4 - 4\cdot e \\ 4\cdot e \\ 0 \\ 1 \end{array}\right]$$

EXERCISE 2(a)

#25: $$GRAD\left(\left[z\cdot LOG(x + y) + x\cdot e^{x^{2}\cdot y}\right], [x, y, z]\right)$$

#26:

$$\left[\begin{array}{c} e^{x^{2}\cdot y}\cdot(2\cdot x^{2}\cdot y + 1) + \dfrac{z}{x + y} \\ x^{3}\cdot e^{x^{2}\cdot y} + \dfrac{z}{x + y} \\ LN(x + y) \end{array}\right]$$

#27: $$\begin{bmatrix} \frac{2}{5} - 8\cdot e^{-9/2} \\ 27\cdot e^{-9/2} + \frac{2}{5} \\ \mathrm{LN}\left(\frac{5}{2}\right) \end{bmatrix}$$

EXERCISE 2(b)

#28: $$y^2\cdot\sqrt{z} + \frac{x^{5/3}\cdot \mathrm{COS}(z\cdot x)}{e^z}$$

#29: $$\mathrm{GRAD}\left(\left[y^2\cdot\sqrt{z} + \frac{x^{5/3}\cdot \mathrm{COS}(z\cdot x)}{e^z}\right], [x, y, z]\right)$$

#30: $$\begin{bmatrix} e^{-z}\cdot\left(\frac{5\cdot x^{2/3}\cdot \mathrm{COS}(x\cdot z)}{3} - z\cdot x^{5/3}\cdot \mathrm{SIN}(x\cdot z)\right) \\ 2\cdot y\cdot\sqrt{z} \\ - e^{-z}\cdot x^{8/3}\cdot \mathrm{SIN}(x\cdot z) - e^{-z}\cdot x^{5/3}\cdot \mathrm{COS}(x\cdot z) + \frac{y^2}{2\cdot\sqrt{z}} \end{bmatrix}$$

#31: $$\begin{bmatrix} \frac{5\cdot 3^{2/3}\cdot e^{-1}\cdot \mathrm{COS}(3)}{3} - 3\cdot 3^{2/3}\cdot e^{-1}\cdot \mathrm{SIN}(3) \\ -1 \\ - 3\cdot 3^{2/3}\cdot e^{-1}\cdot \mathrm{COS}(3) - 9\cdot 3^{2/3}\cdot e^{-1}\cdot \mathrm{SIN}(3) + \frac{1}{8} \end{bmatrix}$$

EXERCISE 3(a)

#32: $$2\cdot x^2 - x\cdot y + y^2 + 2\cdot z^3 + 6$$

To calculate the hessian matrix, first I calculate the partial derivatives:

#33: $\frac{d}{dx}(2\cdot x^2 - x\cdot y + y^2 + 2\cdot z^3 + 6)$

#34: $4\cdot x - y$

And now the gradient vector of that derivative:

#35: GRAD([4·x - y], [x, y, z])

#36: $\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$

That will be the first column in the hessian matrix. No I do the same with y:

#37: $\frac{d}{dy}(2\cdot x^2 - x\cdot y + y^2 + 2\cdot z^3 + 6)$

#38: $2\cdot y - x$

#39: GRAD([2·y - x], [x, y, z])

#40: $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

And now with respect to z:

#41: $\frac{d}{dz}(2\cdot x^2 - x\cdot y + y^2 + 2\cdot z^3 + 6)$

#42: $6\cdot z^2$

#43: GRAD($\left[6\cdot z^2\right]$, [x, y, z])

#44: $\begin{bmatrix} 0 \\ 0 \\ 12\cdot z \end{bmatrix}$

The results in #36, #40 and #44 are the columns of the hessian matrix. I can copy them in the edition bar, using commas (,) to separate the elements in a column, and semicolon (;) to separate the columns. That is to say, I write in the edition bar: [4, -1, 0;-1, 2, 0; 0, 0, 12·z]. And the results is:

#45: $$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 12\cdot z \end{bmatrix}$$

That is the hessian matrix. Now I replace the point (1,1,1):

#46: $$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 12\cdot 1 \end{bmatrix}$$

#47: $$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

EXERCISE 4

#48: $$\frac{x^2 + y^2}{z\cdot(x + y)}$$

We just need to know the partial derivatives. I can calulate the three at the same time with the gradient vector:

#49: $$\mathrm{GRAD}\left(\left[\frac{x^2 + y^2}{z\cdot(x + y)}\right], [x, y, z]\right)$$

#50: $$\begin{bmatrix} \dfrac{x^2 + 2\cdot x\cdot y - y^2}{z\cdot(x + y)^2} \\ -\dfrac{x^2 - 2\cdot x\cdot y - y^2}{z\cdot(x + y)^2} \\ -\dfrac{x^2 + y^2}{z^2\cdot(x + y)} \end{bmatrix}$$

Now I replace x=5, y=8, z=6:

#51: $$\left[\frac{5^2 + 2\cdot5\cdot8 - 8^2}{6\cdot(5 + 8)^2},\ -\frac{5^2 - 2\cdot5\cdot8 - 8^2}{6\cdot(5 + 8)^2},\ -\frac{5^2 + 8^2}{6^2\cdot(5 + 8)}\right]$$

#52: $$\left[\frac{41}{1014},\ \frac{119}{1014},\ -\frac{89}{468}\right]$$

B'x= 41/1014. That means that if we increase x by 1 unit, the benefit will increase 41/1014 units approximately.
B'y= 119/1014. That means that if we increase y by 1 unit, the benefit will increase 119/1014 units approximately.
B'z= -89/468. That means that if we increase z by 1 unit, the benefit will decrease 89468 units approximately.

So I would suggest to increase the production of chocolate B, as this one has the greatest marginal benefit.

EXERCISE 5

#53: $$\frac{100\cdot x\cdot y}{x\cdot y + u\cdot v}$$

I calculate the current value replacing x=21, y=50, u=10, v=70:

#54: $$\frac{100\cdot21\cdot50}{21\cdot50 + 10\cdot70}$$

#55: 60

So at the moment, 60 units are being sold.

a) As the variations are given in percentages, we are asked to calculate the elasticity of the variable x, so we apply the following formula: Ex= f'x(a)•(a1/f(a)).

The derivative with respect to x is:

#56: $\frac{d}{dx} \frac{100\cdot x\cdot y}{x\cdot y + u\cdot v}$

#57: $\frac{100\cdot u\cdot v\cdot y}{(x\cdot y + u\cdot v)^2}$

We replace the point (x=21, y=50, u=10, v=70):

#58: $\frac{100\cdot 10\cdot 70\cdot 50}{(21\cdot 50 + 10\cdot 70)^2}$

#59: $\frac{8}{7}$

Now, we multiply the previous line by the value of x (21) divided by the value of the function at the point (60):

#60: $\frac{8}{7}\cdot\frac{21}{60}$

#61: $\frac{2}{5}$

As this value is for 1%, we have to calculate the value for 2%, that is, multiply the previous value by 2:

#62: $\frac{2}{5}\cdot 2$

#63: $\frac{4}{5}$

#64: 0.8

So the consumption would increase 0.8% if the temperature increases 2%.

b) Now, we are asked to calculate the elasticity of the variable y, so we do the same as in the previous section.

#65: $\frac{d}{dy} \frac{100\cdot x\cdot y}{x\cdot y + u\cdot v}$

#66: $\frac{100\cdot u\cdot v\cdot x}{(x\cdot y + u\cdot v)^2}$

#67: $\frac{100\cdot 10\cdot 70\cdot 21}{(21\cdot 50 + 10\cdot 70)^2}$

#68: $\frac{12}{25}$

#69: $\frac{12}{25}\cdot\frac{50}{60}$

#70: $\frac{2}{5}$

#71: $\frac{2}{5}\cdot(-25)$

#72: -10

So the consumption would decrease 10% if the average income decreases 25%.

EXERCISE 6

#73: $4\cdot LN(10)$

We calculate the present utility by replacing x=8, y=3, z=2 in #1:

#74: 9.210340371

The approximate value of 4*ln(10) is:

#75: $\frac{d}{dx} \left(LN((x + 2)^2)\cdot\left(\frac{y}{2} - 1\right)\cdot(2\cdot z)\right)$

Now we calculate the derivative with respect to x:

#76: $\frac{2\cdot z\cdot(y - 2)}{x + 2}$

#77: $$\frac{2}{5}$$

And replace x=8, y=3, z=2:

#78: $$\frac{d}{dy}\left(LN((x + 2)^2)\cdot\left(\frac{y}{2} - 1\right)\cdot(2\cdot z)\right)$$

The derivative with respect to y:

#79: $z\cdot LN((x + 2)^2)$

#80: $$2\cdot LN((8 + 2)^2)$$

#81: $4\cdot LN(10)$

#82: $$\frac{d}{dz}\left(LN((x + 2)^2)\cdot\left(\frac{y}{2} - 1\right)\cdot(2\cdot z)\right)$$

And with respect to z:

#83: $(y - 2)\cdot LN((x + 2)^2)$

#84: $$(3 - 2)\cdot LN((8 + 2)^2)$$

#85: $2\cdot LN(10)$

#86: $$\frac{2}{5}\cdot 8 + 4\cdot LN(10)\cdot 3 + 2\cdot LN(10)\cdot(-1)$$

As several variables vary at the same time, we must use the differential: DU=U'x*Var(x)+U'y*Var(y)+U'z*Var(z):

#87: $10\cdot LN(10) + \frac{16}{5}$

#88: $$26.22585092$$

So, the new utillity will be the present utility (which was calculated in #73) plus the variation (calculated in #88):

#89: $14\cdot LN(10) + \frac{16}{5}$

#90: $$35.4361913$$

#91: $$\mathrm{LN}((x+2)^2)\cdot\left(\frac{y}{2}-1\right)\cdot(2\cdot z)$$

EXERCISE 7

#92: $$x\cdot y\cdot e^{2\cdot x\cdot y} + y\cdot x$$

We have to calculate the marginal rate of substitution, so we have to compute the first partial derivates of the function, and then divide both.

#93: $$\frac{d}{dx}\left(x\cdot y\cdot e^{2\cdot x\cdot y} + y\cdot x\right)$$

#94: $$y\cdot e^{2\cdot x\cdot y} + 2\cdot x\cdot y^2\cdot e^{2\cdot x\cdot y} + y$$

#95: $$15\cdot e^{2\cdot 80\cdot 15} + 2\cdot 80\cdot 15^2\cdot e^{2\cdot 80\cdot 15} + 15$$

#96: $$36015\cdot e^{2400} + 15$$

#97: $$\frac{d}{dy}\left(x\cdot y\cdot e^{2\cdot x\cdot y} + y\cdot x\right)$$

#98: $$e^{2\cdot x\cdot y}\cdot(2\cdot x^2\cdot y + x) + x$$

#99: $$e^{2\cdot 80\cdot 15}\cdot(2\cdot 80^2\cdot 15 + 80) + 80$$

#100: $$192080\cdot e^{2400} + 80$$

The MRS is $\partial y/\partial x = -f'x/f'y$. So we calculate –#96/#100:

#101: $$-\frac{36015\cdot e^{2400} + 15}{192080\cdot e^{2400} + 80}$$

#102: $$-\frac{3}{16}$$

This means that if x increases 1 unit, y should decrease 3/16 to keep the same benefit. As the manager wants to decrease x by 16 units, we multiply by –16:

#103: $- \frac{3}{16}\cdot(-16)$

#104: 3

So, if the number of employees decreases 16 units, the number of tables should increase 3 units to keep the same benefit.

EXERCISE 8

#105: $20\cdot x^2 + \sqrt{(5\cdot y)} - \frac{3\cdot z}{2}$

To calculate the value of x, I replace y=5 and z=4:

#106: $20\cdot x^2 + \sqrt{(5\cdot 5)} - \frac{3\cdot 4}{2}$

#107: $20\cdot x^2 - 1$

As the pollution is 319, I solve the equation 20x^2-1=319:

#108: $\text{SOLVE}\left(\left[20\cdot x^2 - 1 = 319\right], [x]\right)$

#109: $[x = -4, x = 4]$

As x is a quantity of input, I consider just the solution x=4. So the present quantities are (x=4, y=5, z=4).

I calculate the derivatives of the pollution function.

#110: $\text{GRAD}\left(\left[20\cdot x^2 + \sqrt{(5\cdot y)} - \frac{3\cdot z}{2}\right], [x, y, z]\right)$

#111: $\begin{bmatrix} 40\cdot x \\ \frac{\sqrt{5}}{2\cdot\sqrt{y}} \\ -\frac{3}{2} \end{bmatrix}$

I replace the present point:

#112: $\left[\begin{array}{c} 40\cdot 4 \\ \dfrac{\sqrt{5}}{2\cdot\sqrt{5}} \\ -\dfrac{3}{2} \end{array}\right]$

#113: $\left[\begin{array}{c} 160 \\ \dfrac{1}{2} \\ -\dfrac{3}{2} \end{array}\right]$

So, if x increases 1 unit, the pollution increases 160 units (approximately).
If y increases 1 unit, the pollution increases 1/2 units (approximately).

Now we use the production function:

#114: x·y·z

#115: GRAD([x·y·z], [x, y, z])

#116: $\left[\begin{array}{c} y\cdot z \\ x\cdot z \\ x\cdot y \end{array}\right]$

#117: $\left[\begin{array}{c} 5\cdot 4 \\ 4\cdot 4 \\ 4\cdot 5 \end{array}\right]$

#118: $\left[\begin{array}{c} 20 \\ 16 \\ 20 \end{array}\right]$

If x increases 1 unit, the pollution increases 20 units (approximately).
If y increases 1 unit, the pollution increases 16 units (approximately).

EXERCISE 9

#119: $\frac{d}{dx}\left(40\cdot x + 1100\cdot z + z\cdot\left(15\cdot y + 10\cdot x - \frac{x\cdot y}{2}\right)\right)$

We have to use the differential because two of the variables show changes. So, we have to calculate the first partial derivate of each one at the current point and multiply each one by the variation suffered by each variable.

#120: $-\frac{y\cdot z - 20\cdot z - 80}{2}$

#121: 40

We replace the point (x=10, y=20, z=7).

#122: $\frac{d}{dz}\left(40\cdot x + 1100\cdot z + z\cdot\left(15\cdot y + 10\cdot x - \frac{x\cdot y}{2}\right)\right)$

var(x)=-1.5

#123: $x\cdot\left(10 - \frac{y}{2}\right) + 15\cdot y + 1100$

#124: 1400

We replace the point (x=10, y=20, z=7).

#125: $40\cdot(-1.5) + 1400\cdot 2$

Var(z)=2

Now, we apply the following expression: Df(a)= f'x(a)*Var(x)+ f'y(a)*Var(y)+f'z(a)*Var(z). (Var(y)=0, so the second addend is 0):

#126: 2740

#127: $40\cdot x + 1100\cdot z + z\cdot\left(15\cdot y + 10\cdot x - \frac{x\cdot y}{2}\right)$

So the income will increase 2740 unit approximately.

EXERCISE 10

#128: $\frac{5}{\sqrt{r}}\cdot\left(\frac{r}{10}\cdot\sqrt{r} + 3\cdot LOG(p)\right)$

#129: $\frac{d}{dp}\left(\frac{5}{\sqrt{r}}\cdot\left(\frac{r}{10}\cdot\sqrt{r} + 3\cdot LOG(p)\right)\right)$

#130: $$\frac{15}{p\cdot\sqrt{r}}$$

The price-elasticity is the derivative multiplied by the variable and divided by the function:

#131: $$\frac{\frac{15}{p\cdot\sqrt{r}}\cdot p}{\frac{5}{\sqrt{r}}\cdot\left(\frac{r}{10}\cdot\sqrt{r} + 3\cdot LOG(p)\right)}$$

#132: $$\frac{30}{30\cdot LN(p) + r^{3/2}}$$

I replace the values p=200, r=2000:

#133: $$\frac{30}{30\cdot LN(200) + 2000^{3/2}}$$

#134: $$\frac{3}{3\cdot LN(200) + 4000\cdot\sqrt{5}}$$

#135: 0.0003348151933

So, the demand would increase 0.00033% approximately if the price would increase 1%. It is a demand quite inelastic.

Now with respect to the income:

#136: $$\frac{d}{dr}\left(\frac{5}{\sqrt{r}}\cdot\left(\frac{r}{10}\cdot\sqrt{r} + 3\cdot LOG(p)\right)\right)$$

#137: $$\frac{1}{2} - \frac{15\cdot LN(p)}{2\cdot r^{3/2}}$$

The income-elasticity is the derivative multiplied by the variable and divided by the function:

#138: $$\frac{\left(\frac{1}{2} - \frac{15\cdot LN(p)}{2\cdot r^{3/2}}\right)\cdot r}{\frac{5}{\sqrt{r}}\cdot\left(\frac{r}{10}\cdot\sqrt{r} + 3\cdot LOG(p)\right)}$$

#139: $$\frac{r^{3/2} - 15\cdot LN(p)}{30\cdot LN(p) + r^{3/2}}$$

I replace the values p=200, r=2000:

#140: $$\frac{2000^{3/2} - 15\cdot LN(200)}{30\cdot LN(200) + 2000^{3/2}}$$

#141: $$\frac{8000\cdot\sqrt{5} - 3\cdot LN(200)}{2\cdot(3\cdot LN(200) + 4000\cdot\sqrt{5})}$$

#142: $$0.9973390642$$

So, the demand would increase 0.997% approximately if the income would increase 1%. It is a normal good, as the demand increases when the income increases.

EXERCISE 11

#143: $$LN(x)^{2\cdot t - 4\cdot z\cdot v} - \frac{v\cdot e^{x\cdot y\cdot t}}{t\cdot z\cdot LN(v\cdot e^{x\cdot y + 2\cdot t})}$$

#144: $$GRAD\left(\left[LN(x)^{2\cdot t - 4\cdot z\cdot v} - \frac{v\cdot e^{x\cdot y\cdot t}}{t\cdot z\cdot LN(v\cdot e^{x\cdot y + 2\cdot t})}\right], [x, y, z, t, v]\right)$$

#145:

$$\frac{(2\cdot t - 4\cdot v\cdot z)\cdot LN(x)^{-4\cdot v\cdot z + 2\cdot t - 1}}{x} - \frac{v\cdot y\cdot e^{t\cdot x\cdot y}\cdot(t\cdot L\sim}{t\cdot z\cdot(LN\sim}$$

$$- \frac{v\cdot x\cdot e^{t\cdot x\cdot y}\cdot(t\cdot LN(v) + t\cdot x\cdot y + 2\cdot t\sim}{t\cdot z\cdot(LN(v) + x\cdot y + 2\cdot t)^{2}}$$

$$\frac{v\cdot e^{t\cdot x\cdot y}}{t\cdot z^{2}\cdot(LN(v) + x\cdot y + 2\cdot t)} - 4\cdot v\cdot LN(x)^{2\cdot(t - 2\sim}$$

$$2\cdot LN(x)^{2\cdot(t - 2\cdot v\cdot z)}\cdot LN(LN(x)) - \frac{v\cdot e^{t\cdot x\cdot y}\cdot((t\cdot x\cdot y - 1)\cdot LN(v) + \sim}{t^{2}\cdot z\cdot(LN(v)\sim}$$

$$- 4\cdot z\cdot LN(x)^{2\cdot(t - 2\cdot v\cdot z)}\cdot LN(LN(x)) - \frac{e^{t\cdot x\cdot y}\cdot(LN(v\sim}{t\cdot z\cdot(LN(v\sim}$$

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─────────────────────────
                2
(2) + 2·2 + 2·2)

 - 1)
──────



·2·2)
     ·LN(LN(2))


   2  2           2
2·2 ·2  + 2·2·(2·2  - 1) - 4·2)
────────────────────────────────
            2
+ 2·2 + 2·2)

) + 2·2 + 2·2 - 1)
───────────────────
              2
) + 2·2 + 2·2)
```

#147:

$$\begin{bmatrix} -1350.125006 \\ -646.3720106 \\ 324.1093941 \\ -620.2403118 \\ 162.5163914 \end{bmatrix}$$