19 a) $f(0,0,0,0) = (-1,0) \neq \vec{0}$

$f$ is not linear because $f(\vec{0}) \neq \vec{0}$

b) $f(x,y) = y^2 - x$

It's not linear because it is not a one degree polynomial. I will show which property is not verified:

$\vec{u_1} = (1,1)$ ; $f(\vec{u_1}) = 0$

$\vec{u_2} = (2,3)$ ; $f(\vec{u_2}) = 9-2 = 7$

$f(\vec{u_1}) + f(\vec{u_2}) = 0 + 7 = 7$

$\vec{u_1} + \vec{u_2} = (3,4)$ ; $f(\vec{u_1} + \vec{u_2}) = 16 - 3 = 13$

$f(\vec{u_1} + \vec{u_2}) \neq f(\vec{u_1}) + f(\vec{u_2})$

$\Rightarrow$ $f$ doesn't preserve the addition of vectors.

c) $f(x,y) = (y,0)$ is linear.

$f(1,0) = (0,0)$

$f(0,1) = (1,0)$

$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

d) $f(x,y,z) = (2x, \; y - xz)$ is not linear.

$\vec{u_1} = (1,2,3)$ ; $f(\vec{u_1}) = (2, -1)$

I now consider the scalar $\alpha = 3$, for example.

$\alpha \cdot f(\vec{u_1}) = 3 \cdot (2,-1) = (6, -3)$

$\alpha \cdot \vec{u_1} = 3\,(1,2,3) = (3,6,9)$

$f(\alpha \vec{u_1}) = f(3,6,9) = (6, 6-18) = (6,-12)$

$f(\alpha \vec{u_1}) \neq \alpha \cdot f(\vec{u_1})$

$\Rightarrow$ $f$ doesn't preserve the product of a vector by a scalar $\Rightarrow$ $f$ is not a linear map.

e) $f(x,y,z) = (2x, \; y - z)$

$f(1,0,0) = (2,0)$

$f(0,1,0) = (0,1)$

$f(0,0,1) = (0,-1)$

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

f) $f(x,y) = (x+1, 2y, x+y)$

$f(0,0) = (1,0,0) \neq \vec{0}$

$f$ is not linear

g) $f(x,y) = \begin{vmatrix} x & y \\ 1 & 2 \end{vmatrix} = 2x - y$ is linear.

$f(1,0) = 2$

$f(0,1) = -1$

$A = (2 \quad -1)$ is the associated matrix.