

Mathematics for Business II

2nd G. A. D. E. – 2012/13 Academic Year

Exercises Unit 3 – Linear Programming

1. A company has the possibility of manufacturing three different products A , B and C using for it a specific machine that functions 45 hours per week. A unit of item A leaves a net profit of 4 m.u., article B 12 m.u., and C , 3 m.u. The output of the machine for the three products is 50, 25, and 75 units, respectively, per product/hour. That is, manufacturing only product A , the machine can produce 50 units in 1 hour and equivalently in respect of the other products. According to a market study, the sales possibilities are no more than 1 000 units of A , 5 000 of B and 1 500 of C per week. Formualte a mathematical problem that makes it possible to plan production so as to obtain the greatest possible profit.

2. Three bulldozers A , B and C can extract a maximum of 200, 500, and 300 Mt of three minerals, A , B and C respectively (bulldozer A extracts mineral A , bulldozer B extracts mineral B and bulldozer C extracts mineral C). Daily production is stored firstly in a site with a maximum capacity of $1\,800\text{ m}^3$. The specific volumes of minerals A , B and C are 1.8, 2 and $2.2\text{ m}^3/\text{Mt}$ respectively. The minerals are washed on the following day. The washing facility is able to wash 80, 90 and 100 Mt per hour, respectively, and the working hours of the mineral washing facility are limited to 10 h. The profit per unit is 4, 5 and 6 monetary units per ton, respectively. Formulate the problem to find the best breakdown of the amounts to extract.

3. A plant manufactures four different products (A , B , C , D), using four different machines (M1, M2, M3, M4). The following table shows the number of minutes required for each machine to work in order to produce one kilogram of each product as well as the maximum weekly demand (in kg) for A , B , C and D .

<i>Product</i>	<i>M1</i>	<i>M2</i>	<i>M3</i>	<i>M4</i>	<i>Maximum demand</i>
A	5	–	6	–	100
B	3	6	4	3	400
C	4	5	3	3	450
D	4	2	–	2	200

The availability of each machine is 60 hours per week. Products *A*, *B*, *C* and *D* are sold at 9, 7, 6 and 5 € per kilo, respectively. The variable costs of manpower are 2€ per hour for machines *M1* and *M2*, and 3€ per hour for machines *M3* and *M4*. The cost of materials for each kilogram of the product are 4€ for product *A*, and 1€ for the other products (*B*, *C* and *D*). Formulate the problem to maximise the company's profit.

4. The company Transvaal extracts diamonds from three South African mines by processing the land obtained from the mine. The capacity, number, weight of the gems and costs vary at each mine. These data are reflected in the following table:

	Capacity (m ³ of land that can be processed)	Treatment cost of the gems (€/m ³)	Purity (carats/m ³)	Number of gems per m ³
Mine 1	83,000	0.60	0.36	0.58
Mine 2	310,000	0.36	0.22	0.26
Mine 3	190,000	0.50	0.36	0.21

Based on marketing considerations, the minimum required monthly production is 148,000 gems. Another requirement is that the production's purity is at least 130,000 carats. Furthermore, in accordance with an agreement with local authorities, the volume of processed earth in mine II cannot exceed more than 60% of the combined amount of land processed in mines I and III together. Formulate this situation as a linear programming problem.

5. The town hall of Cartagena is considering building in a 4 hectare (ha) plot of land outside the city. The plans are to build two types of houses: subsidised and at the appraisal price. The building cost of a subsidised house is 130,000 € and 120 houses of this type can be built in each hectare of land, while the cost of the houses at the appraisal price is 180,000 € and 90 houses can be built per hectare. The estimated

potential market is for a total of 400 homes. The bank extending the mortgage loan for all the houses is not willing to lend more than 60 million Euros and the town hall requires that the subsidised houses account for at least 65% of the homes built.

Formulate the problem for the two following cases:

- a) The aim is to maximise the total number of houses to be built.
- b) The aim is to minimise the total building costs of the houses.

6. Solve the following problem graphically:

$$\begin{aligned}
 \text{Min } z &= 3x_1 + 2x_2 \\
 \text{s. t. } 2x_1 + x_2 &\geq 10 \\
 -3x_1 + 2x_2 &\leq 6 \\
 x_1 + x_2 &\geq 6 \\
 x_1, x_2 &\geq 0
 \end{aligned}$$

7. Solve the following problem graphically:

$$\begin{aligned}
 \text{Min } & x_1 + x_2 \\
 \text{s. t. } & x_1 + x_2 \geq 1 \\
 & 4x_1 + 2x_2 \leq 6 \\
 & x_1 \geq 0, \quad x_2 \geq 0
 \end{aligned}$$

8. Solve the following problem graphically:

$$\begin{aligned}
 \text{Max } & 2x_1 + 5x_2 \\
 \text{s. t. } & x_1 + x_2 \geq 4 \\
 & x_1 \geq 2 \\
 & x_1 \geq 0, \quad x_2 \geq 0
 \end{aligned}$$

9. Solve the following linear programming problem:

$$\begin{aligned}
 \text{Max } & 4x_1 + 3x_2 + 6x_3 \\
 \text{s. t. } & 3x_1 + x_2 + 3x_3 \leq 30 \\
 & 2x_1 + 2x_2 + 3x_3 \leq 40 \\
 & x_i \geq 0
 \end{aligned}$$

10. Consider the following feasible region and solve the problem with the objective indicated in each section:

$$-10x_1 + 10x_2 \leq 20$$

$$15x_1 - 30x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

- a) $\text{Max } 3x_1 - 3x_2$
- b) $\text{Min } 3x_1 - 3x_2$
- c) $\text{Max } 3x_1 + 3x_2$
- d) $\text{Min } 3x_1 + 3x_2$

11. A transport firm in Cartagena (Spain) exports fruit within the European Union. To this end, it has 5 type *A* trucks (small), 4 type *B* trucks (medium) and 2 type *C* trucks (large). The load capacity (in *Mt*) that each truck is able to transport and the consumption (in litres of diesel fuel per each 100 km) for each type of truck (when carrying a maximum load) are as shown in the following table:

	Load (<i>Tm</i>)	Consumption (l/100 km)
A	20	15
B	30	20
C	45	25

An order came this morning for transporting a shipment of fruit: 85 *Mt* to Milan (1800 km from Cartagena), 60 *Mt* to Bruges (2100 km) and 95 *Mt* to Budapest (3600 km). Assuming that the truck making the trip must carry the maximum load (it does not matter if the merchandise transported exceeds demand at the destination since, in that case, it is stored for subsequent use), formulate a problem for planning the best distribution of trucks per destination in order to minimise the total cost of fuel.

12. *Gran Club de Jugadores* offers trips from Cartagena to the casinos located in the Cote d'Azur. The Club's weekly budget for promoting these trips is 800 Euros. This money can be spent in four types of advertisements: local TV, newspaper, and in two types of radio spots (short and long). The company's target is to reach the maximum potential audience through the different media.

The following table shows the number of potential players the company expects to gain by placing an advertisement in each of the four medium as well as the cost per ad and the maximum number of ads that each medium allows per week.

Media	Audience per ad	Cost per ad	Maximum no. ads per week
TV (1 min.)	5000	80 €	12
Newspaper (1 page)	8500	92 €	5
Radio (short)	2400	29 €	25
Radio (long)	2800	38 €	20

A minimum of 5 spots, between short and long, is required by the radio stations. Moreover, to diversify its campaign, the company decides not to spend more than 180 Euros in radio spots. Pose a linear problem for this situation.

13. A teleshopping firm's work schedule is from 9:00 am to 5:00 pm. Its sales staff requirements vary per hour and its minimum requisites are the following:

Hour	Minimum no. shop assistants
09-10	10
10-11	12
11-12	14
12-13	16
13-14	18
14-15	17
15-16	15
16-17	10

The company can have no more than 12 full-time employees working from 9:00 am to 5:00 pm, with one hour for lunch. Half of them have lunch from 11:00 am to noon and the other half from noon to 1:00 pm. Part-time employees work four hours straight and can start their workday at any hour between 9:00 am and 1:00 pm and are not entitled to a break. As per the company's policy, the number of part-time workers cannot be more than 50% of the total number of workers.

Part-time workers earn 8 Euros per hour (32 Euros per day), whereas those who work full time earn 50 Euros per day. Formulate a problem to plan the optimal hiring with the least personnel expenses.

14. A vendor at indoor soccer games played at the municipal sports pavilion in Cartagena goes around with a tray selling savoury pies, bottles of water, beer, and soft

drinks. The vendor has many years of experience and has noticed that he is able to sell all the goods that he carries provided the following conditions are met:

- At least half of the beverages must be bottles of beer.
- The number of soft drinks must be at least 15% less than those of beer.
- The number of bottles of water cannot exceed one fourth of the total beverages.
- There must be at least 5 units of each beverage.
- The number of savoury pies must be at least half the number of bottled water but never more than 75% of the total number of beverages.

The purchase and sales retail prices (in Euros) for each item are detailed in the following table. To know the number of items that the vendor is able to carry, the number of units of a product that fit in a tray, as if this was the only product to be carried, was measured. These data are reflected in the last column in the table.

<i>Product</i>	<i>Purchasing price</i>	<i>Selling price</i>	<i>Units in a tray</i>
Beer can	0.33	1	45
Soft drink can	0.27	1	45
Water bottle	0.12	0.5	60
Pie	0.30	1	25

Formulate the problem so that the vendor obtains the maximum profit assuming that he is able to sell everything that he carries on the tray.

15. A farmer owns a 640 m^2 plot of land for growing fruit trees: orange, pear, and apple trees. The question is how to divide the plot among the three varieties to achieve the maximum profit knowing that:

- Each orange tree requires 16 m^2 , each pear tree 4 m^2 and each apple tree 8 m^2 .
- He employs 4 workers, each of whom works 150 hours a year. Each orange tree requires 30 manpower hours per year, each peach tree 5 hours, and each apple tree 10 hours.
- The profit per unit is 50, 25 and 20 Euros per orange tree, pear tree and apple tree respectively.

16. A winemaker bottles and sells three types of wine, *Marqués de Mer Luzón*, *Señorío de Colo Cón*, and *Conde Lope León*, obtaining a profit per bottle of 1.25, 2.5 and 2 Euros respectively. All the bottles go through two phases: filling and sealing. The first phase involves 4 hours maximum per day whereas the second phase is programmed to work exactly 6 hours a day. The time (in minutes) that each bottle requires in each phase is shown in the following table:

	Filling time (min.)	Sealing time (min.)
<i>Marqués</i>	1	2
<i>Señorío</i>	2	4
<i>Conde</i>	3	3

Determine all the combination of bottles of wine that the winery can produce in order to maximise its profit. What would be his maximum daily profit?

17. A manufacturing firm eliminated production of a certain line of unprofitable products, creating a considerable surplus production capacity. Management want to use this capacity to produce another three products which we will call A, B and C. Three machines are involved in the production of these products and their weekly available capacity is as shown in the table below:

Machine	Available hours/week
M1	500
M2	300
M3	200

The number of hours-machine required to produce each product unit is:

Machine	Product A	Product B	Product C
M1	8	4	2
M2	5	4	0
M3	3	0	2

The sales department has indicated that the potential sales of product C are not more than 20 units per week. The profit per unit of each product is 50, 45 and 25 Euros, respectively. Determine the following data:

- Which products and in what quantities should the company manufacture.
- The profit to be obtained from manufacturing the above-mentioned products,

broken down by type of product.

- c) The number of hours per week that each of the three machines would use, indicating the surplus hours in any of these machines.
- d) If machine M1 is available 1 more hour, which would be the profit? If machine M2 is available 1 more hour, which would be the profit? If machine M3 is available 1 more hour, which would be the profit?
- e) What happens if the unitary profit of product A varies? And if the unitary profit of product B varies? And if the unitary profit of product C varies?

18. *ICT* is a group that invests in company loans, Treasury Bonds, gold, and real estate. In order to diversify its portfolio, management has established limits on the sums that can be invested in each type of product. *ICT* has a total of 5 million Euros to invest and must comply with two conditions:

- (a) At least 55% of invested sums must be earmarked for gold and for real estate investments.
- (b) No less than 15% of investment must be spent in corporate loans.

The interest rate for each type of investments and the maximum permitted sums are detailed in the following table:

Sector	Interest	Maximum investment
Company loans	7 %	1.000.000 €
Treasury bonds	5 %	2.500.000 €
Gold	3 %	1.500.000 €
Real estates	12 %	1.800.000 €

ICT's objective is to earn as much total interest as possible on its investments. Pose a linear problem for this situation.

19. The national government is polishing up the final details for a significant investment in public works in order to put a stop to the growing rate of unemployment. Consisting of 10 million Euros, the investment plan considers Spain as divided into 5 major regions: north coast, north-east, central region, south-west, and the Mediterranean coast. An estimate has been made in each of these regions on the relationship between each million Euros invested and the number of people who would keep their jobs.

Investment	No. jobs kept/million Euros invested
North Coast	450
North-east	420
Central	505
South-west	610
Mediterranean	470

The Government also wants to define the breakdown of investments so as to maintain as many jobs as possible but requires compliance with the following political considerations:

- Each region must receive at least 1 billion Euros in investments.
- The regions with fewer infrastructures are those in the South-west and in the North coast, so at least 50% of total investment must be divided between these two regions.
- Those regions which are politically closest to the Government are the South-East and the North-East, while those which are not as close are the Central region and the Mediterranean coast. These investments must be distributed ensuring that the total received by the region closest to the Government is not less than the total received by the regions not as close.
- The Government wants to allocate the largest investments to the North-east region.
- Pursuant to law, the money invested in the North coast must not be more than 20% below the sum invested in the North-east.

Formulate a linear problem which helps the Government to decide the best investment plan following these requirements.

20. A manufacturing company of monitors has to decide the number of 22 and 19-inch monitors it has to produce. A market study indicates that it is possible to sell more than 40 units of the 22-inch and exactly 20 of the 19-inch monitors on a daily basis. The maximum number of available manpower hours is 500 h per day. A 22-inch monitor requires 20 hours of work and its sale generates a profit of 120 Euros. Each 19-inch monitor requires 10 hours of work and generates a profit of 80 Euros. A retailer agrees to buy all the monitors manufactured provided that the numbers do not exceed the

maximum according to the market study.

Formulate this situation and solve it graphically. Answer the following questions:

- How many monitors of each model should the company produce every day?
- What would be the company's daily profit?
- How many manpower hours would the company use on a daily basis?

21. A textile firm produces 5 types of fabric: Aranjuez, Bordeaux, Lisbon, Teruel, and Toledo. These are manufactured at the firm's looms using two types of threads: normal and gold. Each of the 5 types of fabric requires different processing time at the looms and different amounts of the two types of thread. The following table details the amounts of normal thread, gold thread (in metres) and processing time at the looms (measured in manpower hours) required to manufacture 1 metre of fabric, as well as the estimated profit per meter for each type of fabric. The number of manpower hours of the looms and the amounts of thread are limited per week as indicated in the table.

	Normal thread (m)	Gold thread (m)	Looms (h)	Profit (€/m)
<i>Aranjuez</i>	0.30	0.20	0.40	2.5
<i>Bordeaux</i>	0.20	0.45	0.30	3.8
<i>Lisbon</i>	0.10	0.50	0.45	4.2
<i>Teruel</i>	0.05	0.20	0.15	1.5
<i>Toledo</i>	0.50	0.10	0.25	2.0
Availability	350	300	160	

Answer the following questions:

- What amount of each fabric should the company produce each week? What is the maximum weekly profit?
- What amount of thread will the company use each week? Will there be any of the two types of thread left over? How much?
- What amount of manpower hours at the looms will the company use each week?
- At present, 1 m of gold thread costs 1 Euro. Would you suggest to pay for 1 meter more of this thread?
- Which is the maximum price for an additional hour of loom that should be advisable to pay?
- The profit of the Bordeaux fabric has fallen to 3 Euros. Should the firm change its production?

22. An investor who wants to invest in listed securities has noticed that there are currently three interesting sectors: Electrical utilities, with an approximate return of 4%, Banks with an approximate return of 5%, and Services, with a 9% return.

He has 2 million Euros and an expert investor has advised him not to invest in Electrical utilities more than half of the total amount invested, and to invest at least 700 000 Euros in banks. Since the shares in the Service sector entail greater risk, the advisor also recommends that the investment in this sector should not be more than 400 000 Euros. Assuming that he invests all of his available capital, explain how the investment should be broken down in order to obtain the maximum total return.

23. A family-owned company engages in raising pigs for their sale and wants to determine the amounts available of different types of foodstuff (corn, flour, and vegetables) that it should feed to the pigs daily. Since pigs eat almost any type of combination of these foods, the goal is to determine which combination complies with certain nutritional requirements (as recommended by a veterinary) at a minimum cost. The following table details the units of each type of basic nutritional ingredients (hydrocarbons, proteins and vitamins) contained in 1 kg of each type of food and the minimum daily amounts and cost of the foodstuff.

Nutritional ingredient	<i>Corn</i>	<i>Flour</i>	<i>Vegetables</i>	<i>Min. amount</i>
Hydrocarbons	90	20	40	200
Proteins	30	80	60	180
Vitamins	10	20	60	150
Cost (€)	0.84	0.72	0.60	

This means, for example, that in 1 kg of corn 90 units of hydrocarbons are found, 30 units of proteins and 10 units of vitamins. The cost of 1 kg of corn is 0.84€. And at least 200 units of hydrocarbons must be included in the daily food of one pig. With these data, how many kg of corn, flour and vegetables must be used to feed a pig every day with the minimum cost? Pose this situation as a linear programming problem and answer the following questions:

- How many amounts of each of the different types of foodstuff should be fed to each pig?
- Is there an alternative combination of food at a minimum cost? Provide a reasoned answer.

- c) Assuming that the company has 100 pigs, how much should be spent each day to feed all the animals?
- d) What are the amounts of hydrocarbons, proteins, and vitamins that each of the pigs will consume on a daily basis?
24. The simplex algorithm has been applied to a certain linear problem and three optimal solutions have been reached $X_1=(2,1,0,5)$, $X_2=(3,0,2,4)$ and $X_3=(0,-6,3,3)$.
- a) Determine all the optimal solutions.
- b) Determine the optimal solution or solutions with the highest first coordinate.
- c) Determine the optimal solution or solutions the first three coordinates of which are equal.
25. The simplex algorithm has been applied to a certain linear problem and three optimal solutions have been reached $(1,5,0,0)$, $(0,2,2,0)$ and $(7,0,1,0)$.
- a) Determine all the optimal solutions.
- b) Is point $(0,2,2,2)$ an optimal solution to the problem?
- c) Determine, among the optimal solutions, the first coordinates of which are greater or equal to two.
26. The firm *Cartagena Moda S.A.* engages in manufacturing four types of ties. The most expensive tie is 100% silk, the least expensive tie is made with polyester 100%, and there are two other types of ties with a blend of polyester and cotton: Type *Blend 1* is 50% polyester and 50% cotton, and type *Blend 2* is 30% polyester and 70% cotton. The following table shows the cost and availability (for a production period of one month) for the three types of materials used in the production process.

Material	Price per meter	Available amount (meters)
Silk	2.1 €	800
Polyester	0.6 €	3000
Cotton	0.9 €	1600

The firm has entered into agreements with other companies and large department stores

to sell its ties. These agreements stipulate a minimum production of each type of tie that the firm must cover. The firm also knows the monthly demand for each type of tie and plans production to avoid having surplus production. The following table summarises the retail price per tie, the minimum production foreseen in each contract, monthly demand, the material required for manufacturing one tie (in metres) and the composition of each tie.

Type of tie	Selling price (€)	Minimum monthly production (units)	Maximum monthly demand (units)	Material used in one tie (metres)	Composition
<i>Silk</i>	6.70€	6000	7000	0.125 m.	100 % Silk
<i>Polyester</i>	3.55€	10000	14000	0.080 m.	100 % Polyester
<i>Blend 1</i>	4.31€	13000	16000	0.100 m.	50 % Polyester 50 % Cotton
<i>Blend 2</i>	4.81€	6000	8500	0.100 m.	30 % Polyester 70 % Cotton

Cartagena Moda S.A. wants to maximise profit considering, among the expenses, only the cost of the materials. Formulate a linear problem for this situation.

27. A company manufactures two different products, called *A* and *B*, in three of its own factories, which we will call F1, F2 and F3. Revenue per kilogram of product *A* and *B* are 12 and 10€, respectively. The manpower minutes required for producing 1 kg per product as well as the daily manpower hours at each factory are specified in the following table:

	Product A	Product B	Manpower hours
Factory F1	15	15	800
Factory F2	10	15	600
Factory F3	30	10	200

Pursuant to the firm's contracts, it must manufacture at least 50 kg of each product. Provide a reasoned answer to the following questions:

- Describe the firm's best daily production plan.
- Indicate the percentage of total revenue generated by each of the factories.
- How many manpower hours are required and how many would be left over at

each of the factories?

- d) Explain if the best production plan would be the same if the unitary revenue for product A varies.
- e) Explain if the best production plan would be the same if the unitary revenue for product B varies.
- f) What would be the maximum price the firm should pay for 1 minute more of manpower every month?